Course Material
of
ME-313 (Fluid Mechanics and Machinery)
for
ICD (5th Semester)

Prepared By

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Definition: Mechanics

- Mechanics is a science that deals with the study of changes occurred in a body (stationary and moving) when subjected to external/internal forces.
- The branch of mechanics that deals with bodies at rest is called statics while the branch that deals with bodies in motion is called dynamics
- The subcategory fluid mechanics is defined as the science that deals with the behavior of fluids at rest (fluid statics) or in motion (fluid dynamics), and the interaction of fluids with solids or other fluids at the boundaries.
- The study of fluids at rest is called fluid statics. The study of f1uids in motion, where pressure forces are not considered, is called fluid kinematics and if the pressure forces are also considered for the fluids in motion that branch of science is called fluid dynamics

Fluid Mechanics & Fluid Properties

Definition: Fluid

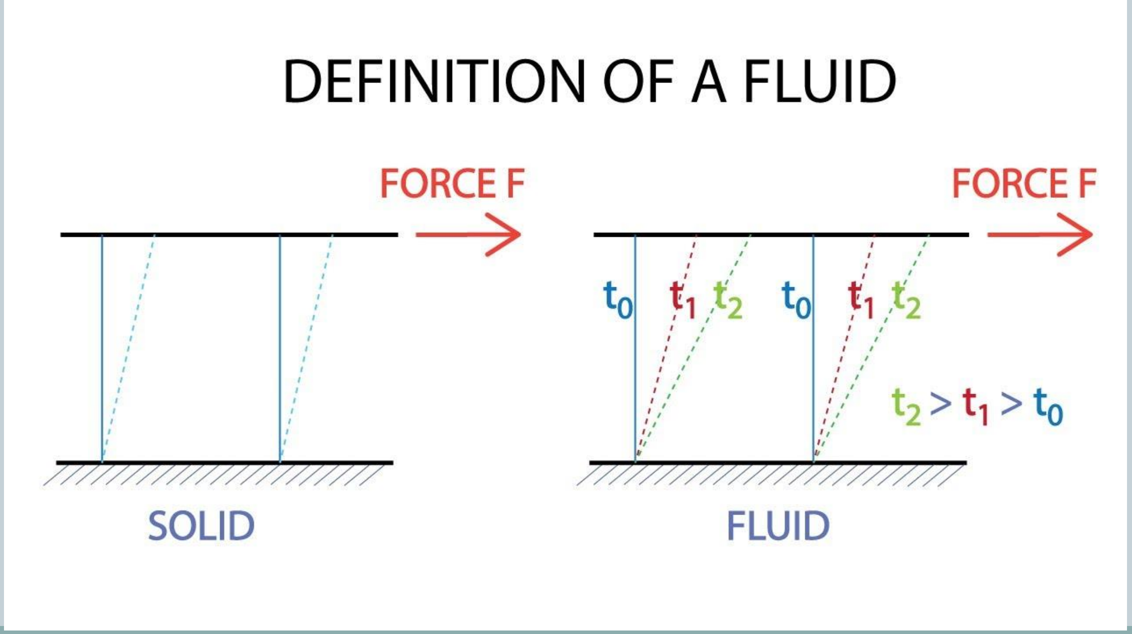
- A substance exists in three primary phases: Solid liquid Gas
- A substance in the liquid or gas phase is referred to as a Fluid.

Difference	
Solid	**Fluid**
Amount of molecules per unit volume are more	Amount of molecules per unit volume are less
Molecular force of attraction is more	Molecular force of attraction is less
Solids have a definite shape	Fluids have no shape
Tensile and compressive: resist	Cannot resist tensile but resist compressive
Solids can sustain a **shear force**; i.e. they remain static	Fluids cannot sustain a **shear force**, i.e. a fluid is always in motion
Stress is a function of the rate of strain.	Stress is a function of strain.

Fluid Mechanics & Fluid Properties

Definition: Fluid

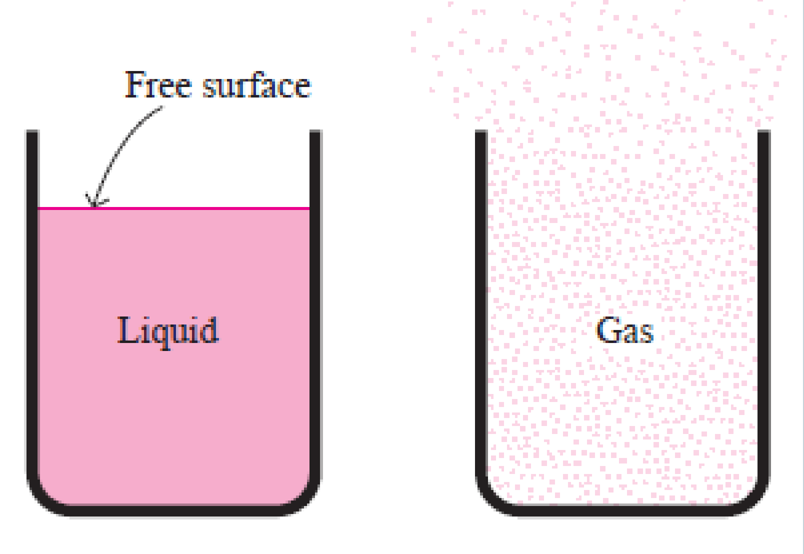
- Fluid: Fluid is a substance that deforms continuously under the action of shear stress, as long as the shear stress is applied, no matter how small the shear stress may be.
- It is this property of fluids not to resist shear stress that makes them capable to flow and take any shape.



Fluid Mechanics & Fluid Properties

Definition: Liquid vs Gasses

- In a liquid, molecules can move relative to each other, but the volume remains relatively constant because of the strong cohesive forces between the molecules.
- As a result, a liquid takes the shape of the container it is in, and it forms a free surface in a larger container in a gravitational field.
- A gas, on the other hand, expands until it encounters the walls of the container and fills the entire available space.
- This is because the gas molecules are widely spaced, and the cohesive forces between them are very small.
- Unlike liquids, gases cannot form a free surface



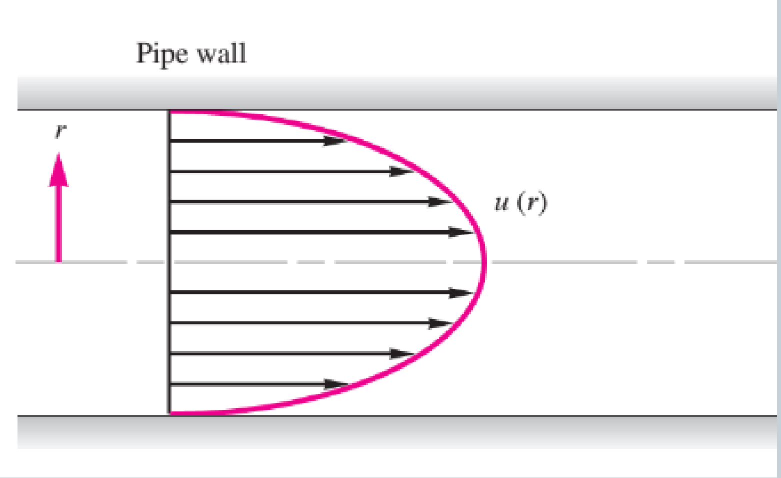
Fluid Mechanics & Fluid Properties

No slip condition

- The layer of the fluid in contact with the solid boundary has the velocity of the boundary itself. In other words, the layer of the fluid sticks to the solid surface in contact and there is "NO SLIP" at the surface.

Lets take a example of pipe

- Inside a pipe or tube a very thin layer of fluid right near the walls of the tube are motionless because they get caught up in the microscopic ridges of the tube. Layers closer to the centre move faster and the fluid sheers. The middle layer moves the fastest.



Specific mass or density ($\boldsymbol{\rho}$)

- The “mass per unit volume” is mass density. Hence it has units of kilograms per cubic meter (kg/m^3).

$$\text{Specific mass or density } (\boldsymbol{\rho}) = \frac{Mass}{Volume} \left\{\frac{kg}{m^3}\right\}$$

- The mass density of water at 4^{o}C is 1000 kg/m^3 while it is 1.20 kg/m^3 for air at 20^{o}C at standard pressure.

Specific weight or weight density (**$\boldsymbol{\omega}$** or **$\boldsymbol{\gamma}$**)

- It is the ratio between the weight of a fluid to its volume. It is also termed as weight per unit volume of a fluid. Its unit is N/m^3.

$$\text{Specific Weight } (\boldsymbol{\omega}) = \frac{Weight}{Volume} = \frac{mass \times gravity}{volume} = \boldsymbol{\rho g} \left\{\frac{N}{m^3}\right\}$$

Problem:

1. What is the specific weight for water at 4^{o}C is ……..
2. What is the specific weight for air ……….

g = 9.81 m/sec^2

Fluid Mechanics & Fluid Properties

Specific volume ($\boldsymbol{v}$)

- It is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume.

$$\text{Specific volume } (\boldsymbol{v}) = \frac{Volume}{Mass} = \frac{1}{\boldsymbol{\rho}} \left\{\frac{m^3}{kg}\right\}$$

Problem:

1. What is the specific volume of water at 4^{o}C is ……..
2. What is the specific volume for air ……….

Fluid Mechanics & Fluid Properties

Specific gravity (S)

- The ratio of specific mass (density) of a given liquid to the specific mass (density) of water (standard fluid) at a standard reference temperature (4^oC) is defined as specific gravity (S).

$$\text{Specific gravity (S)} = \frac{Densisity_{tesing\ fluid}}{Densisity_{standard\ fluid}} \{No\ units\}$$

Example:

- Density of mercury is 13600 kg/m^3. What is the specific gravity.

$$S_{Hg} = \frac{13600}{1000} = 13.6$$

Problem:

1. Find out the specific gravity of oil having density of 800 kg/ m^3?
2. If the specific weight of oil is 8829 N/m^3. Find its specific gravity, density and specific volume?

Fluid Mechanics & Fluid Properties

Viscosity

- Different kinds of fluids flow more easily than others. Water, for example, flows more easily than honey. This is because honey has a higher viscosity.
- Viscosity is a property of fluid by virtue of which a fluid offers resistance to the relative movement of one layer to the other layer adjacent to the first layer.

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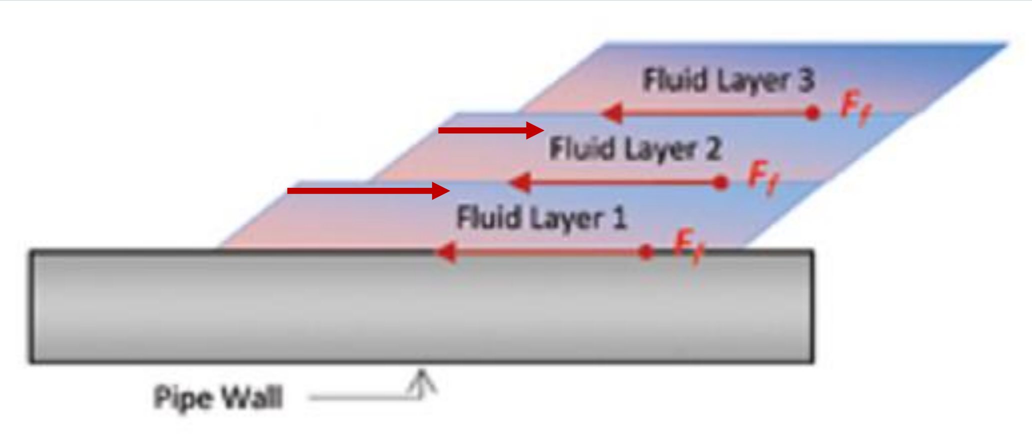
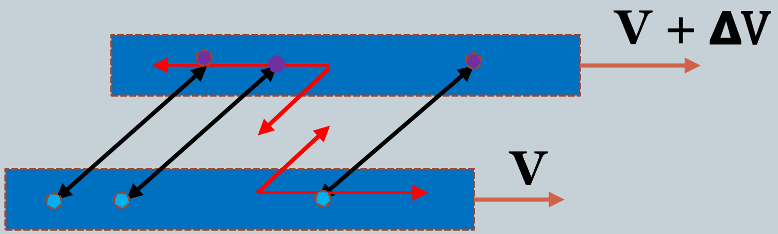


Fluid Mechanics & Fluid Properties

Causes of viscosity

Cohesion force of attraction

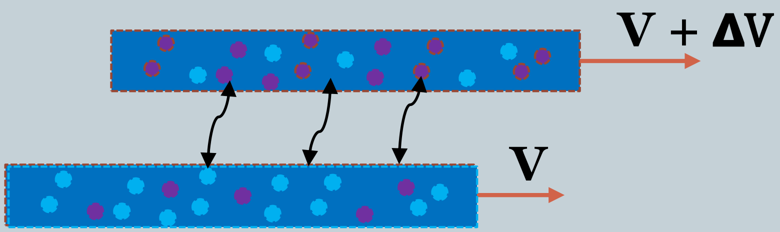
- Particle of a layer attracts the particles of other adjacent layer.



- As a result net force in the backward direction for upper later and forward direction for lower layer. So, the net velocity of upper layer decreases and lower layer increases.

Molecular momentum exchange

- Particles of one layer jumps into the other adjacent layer and vice versa.



- As a result the particles in the upper layer consists of some slower particles and in the lower layer we have some fast moving particles. So, the net velocity of upper layer decreases and lower layer increases.

Fluid Mechanics & Fluid Properties

Newton's Law of Viscosity

By experiment:

$$\left.\begin{array}{l} F \infty A \\ F \infty V \\ F \infty 1/h \end{array}\right\} \quad F \infty \frac{AV}{h} \Rightarrow F = \mu \frac{AV}{h}$$

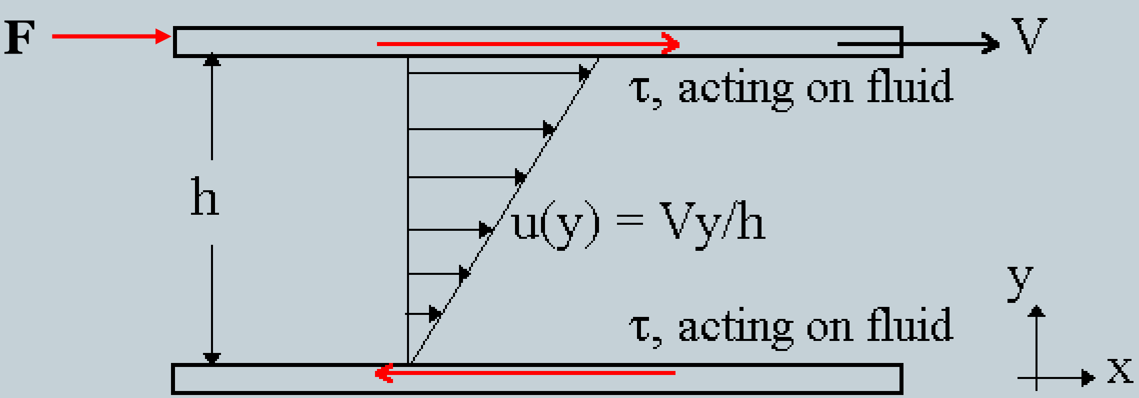
Dynamic viscosity

Also:

$$F = \frac{\mu AV}{h} \qquad \frac{F}{A} = \frac{\mu V}{h}$$

$$\tau = \frac{\mu V}{h} \qquad \tau = \frac{\mu\, u(y)}{y}$$

$$\tau = \frac{\mu\, du}{dy}$$



Area (A)

Fluid Mechanics & Fluid Properties

Newton's Law of Viscosity: It says that the shear stress is proportional to the velocity gradient or shear strain rate.

$$F = \frac{\mu A V}{h}$$

Dynamic viscosity: Units

$$\mu = \frac{Fh}{VA} = \frac{N \times m}{\frac{m}{sec} \times m^2} = \frac{Nsec}{m^2} = \text{Pa-sec}$$

or Units of dynamic viscosity are Poise.

1 Poise = 0.1 Pa-sec

Kinematic viscosity (*ν*) : It is defined as the ratio between the dynamic viscosity and density of the fluid.

$$\nu = \frac{\mu}{\rho} = \frac{\frac{N.sec}{m^2}}{\frac{kg}{m^3}} = \frac{\frac{\left(\frac{kg.m}{sec^2}\right) sec}{m^2}}{\frac{kg}{m^3}} = \frac{m^2}{sec}$$

Fluid Mechanics & Fluid Properties

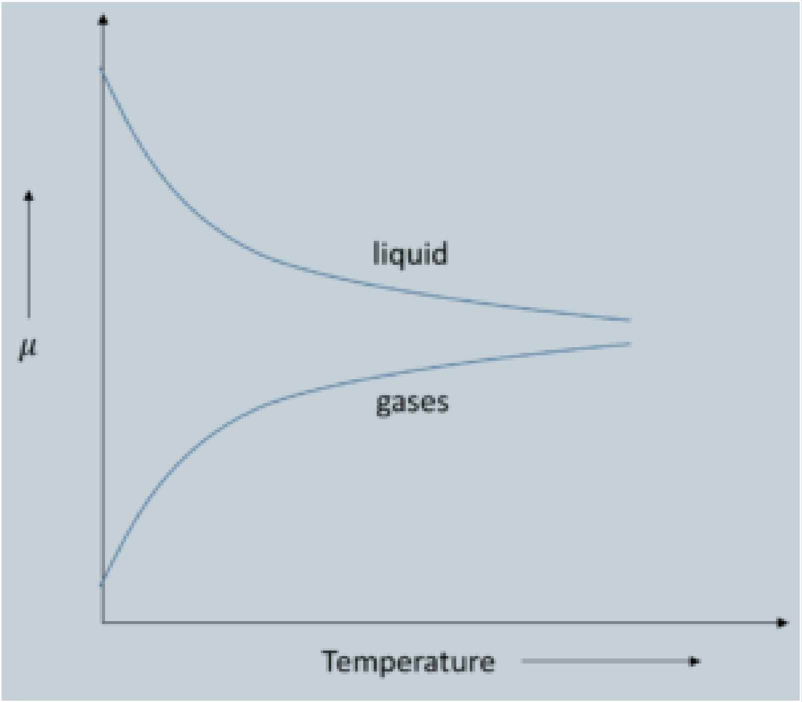
Variation of viscosity with temperature

Liquids

- Viscosity decreases as temperature increases. (with increase in temperature molecules get apart and cohesion force decreases)

Gases

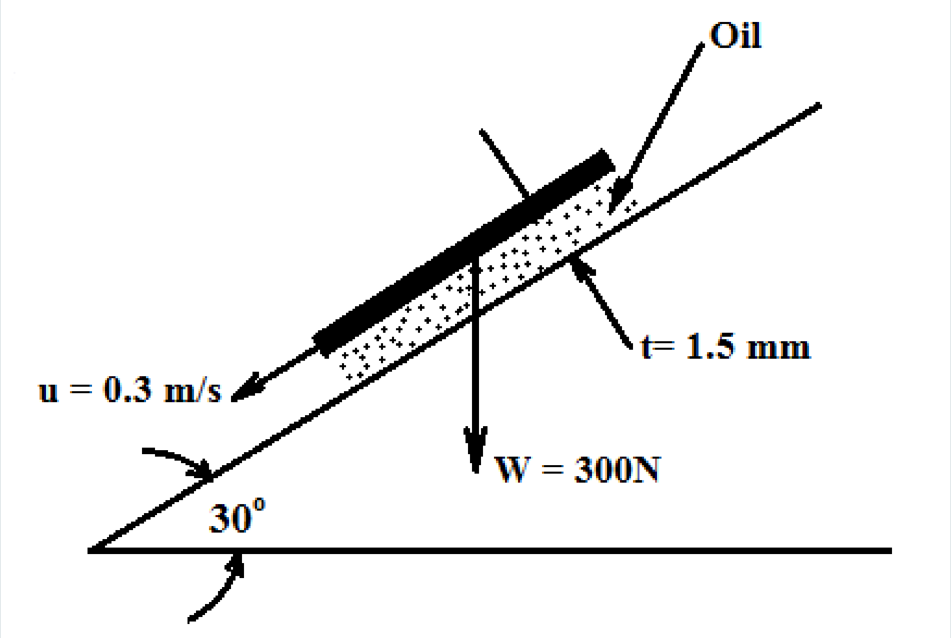
- Viscosity increases as temperature increases. (with increase in temperature intermolecular momentum exchange increases)



Fluid Mechanics & Fluid Properties

Solved Example:

Calculate the dynamic viscosity of an oil, which is used for lubrication between a square plate of size 0 8 m x 0 8 m and an inclined plane with angle of inclination 30^o as shown in Fig. The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0 3 m/s The thickness of oil film is 1.5 mm



Solution. Given :

Area of plate, $A = 0.8 \times 0.8 = 0.64\ \text{m}^2$

Angle of plane, $\theta = 30°$

Weight of plate, $W = 300$ N

Velocity of plate, $u = 0.3$ m/s

Thickness of oil film, $t = dy = 1.5\ \text{mm} = 1.5 \times 10^{-3}$ m

Let the viscosity of fluid between plate and inclined plane is μ.

Component of weight W, along the plane $= W \cos 60° = 300 \cos 60° = 150$ N

Thus the shear force, F, on the bottom surface of the plate $= 150$ N

and shear stress,
$$\tau = \frac{F}{\text{Area}} = \frac{150}{0.64}\ \text{N/m}^2$$

Now using equation (1.2), we have

$$\tau = \mu \frac{du}{dy}$$

where du = change of velocity $= u - 0 = u = 0.3$ m/s

$$dy = t = 1.5 \times 10^{-3}\ \text{m}$$

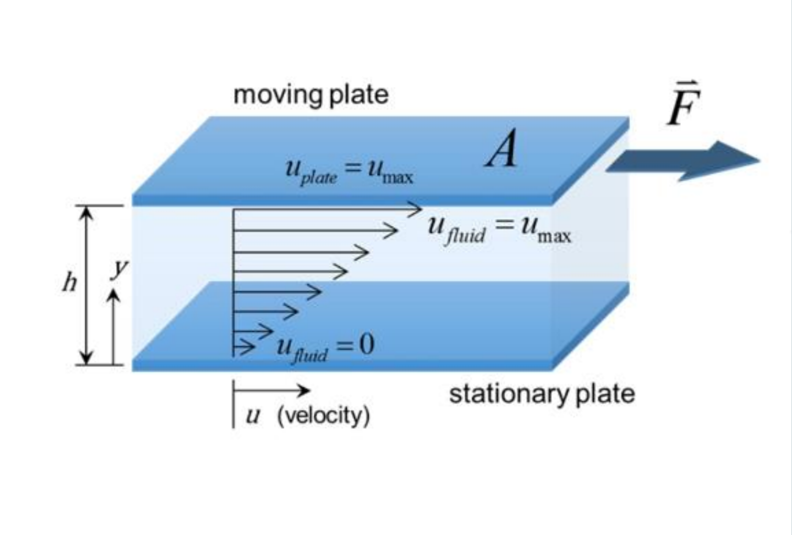
$$\therefore \quad \frac{150}{0.64} = \mu \frac{0.3}{1.5 \times 10^{-3}}$$

$$\therefore \quad \mu = \frac{150 \times 1.5 \times 10^{-3}}{0.64 \times 0.3} = 1.17\ \text{N s/m}^2 = 1.17 \times 10 = \mathbf{11.7\ poise.\ Ans.}$$

Fluid Mechanics & Fluid Properties

Solved Example:

The space between two square flat parallel plates is filled with oil. Each side of the plate is 60 cm. The thickness of the oil film is 12.5 mm. The upper plate, which moves at 2.5 metre per sec requires a force of 98.1 N to maintain the speed.



Determine (i) the dynamic viscosity of the oil, and (ii) the kinematic viscosity of the oil if the specific gravity of the oil is 0.95

Fluid Mechanics & Fluid Properties

Solution. Given:

Each side of a square plate = 60 cm = 0.6 m

Area (A) = 0.6 x 0.6 = 0.36 m^2 . Thickness of oil film (dy) = 12.5 mm = 12.5 x 10^{-3}m

Velocity of upper plate (u) = 2.5 m/s

$\therefore$ Change of velocity between plates, $du = 2.5$ m/sec

Force required on upper plate, $F = 98.1$ N

$\therefore$ Shear stress, $$\tau = \frac{\text{Force}}{\text{Area}} = \frac{F}{A} = \frac{98.1 \text{ N}}{0.36 \text{ m}^2}$$

(*i*) Let μ = Dynamic viscosity of oil

Using equation (1.2), $$\tau = \mu \frac{du}{dy} \quad \text{or} \quad \frac{98.1}{0.36} = \mu \times \frac{2.5}{12.5 \times 10^{-3}}$$

$\therefore$ $$\mu = \frac{98.1}{0.36} \times \frac{12.5 \times 10^{-3}}{2.5} = 1.3635 \frac{\text{Ns}}{\text{m}^2} \quad \textbf{Ans.}$$

Fluid Mechanics & Fluid Properties

Solution. Given:
Each side of a square plate = 60 cm = 0.6 m
Area (A) = 0.6 x 0.6 = 0.36 m^2 . Thickness of oil film (dy) = 12.5 mm = 12.5 x 10^{-3}m
Velocity of upper plate (u) = 2.5 m/s

(*ii*) Sp. gr. of oil, S = 0.95
Let ν = kinematic viscosity of oil
Using equation (1.1 A),

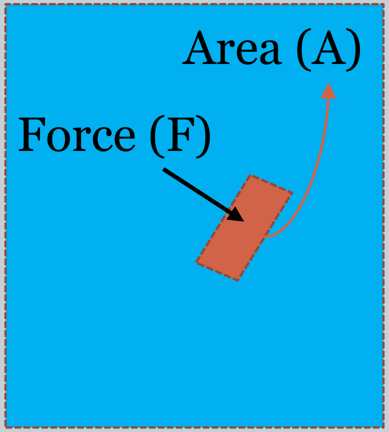
Mass density of oil, $\rho = S \times 1000 = 0.95 \times 1000 = 950 \text{ kg/m}^3$

Using the relation, $\nu = \dfrac{\mu}{\rho}$, we get $\nu = \dfrac{1.3635\left(\dfrac{\text{Ns}}{\text{m}^2}\right)}{950} = .001435 \text{ m}^2/\text{sec}$ **Ans.**

Chapter 2: Static Pressure

Fluid Pressure

- Fluid pressure is defined as a normal force exerted by a fluid on a surface per unit area. The same can be understand from the following figure.
- It has the unit of newtons per square meter (N/m^2), which is called a pascal (Pa).



$$P = \frac{F}{A}$$

The other units for pressure are: Torr, psi, mm of Hg, bar, standard atmosphere, and kilogram-force per square centimeter.

$$1\ \text{Pa} = 1\ \text{N/m}^2$$

$$1\ \text{bar} = 10^5\ \text{Pa} = 0.1\ \text{MPa} = 100\ \text{kPa}$$

$$1\ \text{atm} = 101{,}325\ \text{Pa} = 101.325\ \text{kPa} = 1.01325\ \text{bars}$$

$$1\ \text{kgf/cm}^2 = 9.807\ \text{N/cm}^2 = 9.807\times10^4\ \text{N/m}^2 = 9.807\times10^4\ \text{Pa}$$

$$1\text{atm} = 760\ \text{mm of Hg} = 760\ \text{Torr}$$

$$1\ \text{atm} = 4.696\ \text{psi}$$

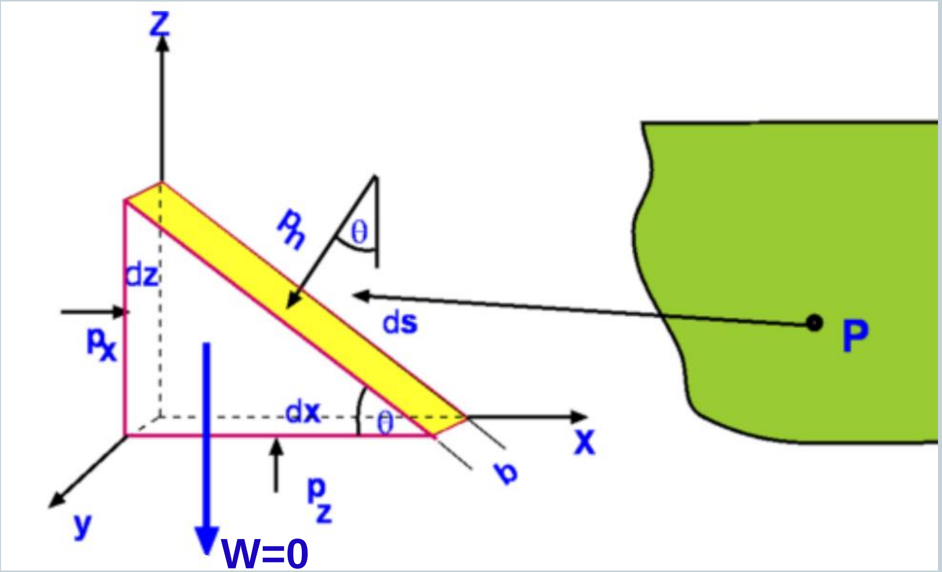
Static Pressure

Pascals Law

- It states that pressure at point in a static fluid is same in all directions.

Proof

- Taking a point P in a static fluid as shown in Figure.
- As the volume of point P is zero so W=0.



As static fluid

$$\Sigma F_x = 0;\ \text{ie.,}\ p_x\, dz\, b \;-\; p_n\, b\, ds\, sin\theta = 0$$

$$\Sigma F_z = 0;\ \text{ie.,}\ p_z\, dx\, b \;-\; p_n\, b\, ds\, cos\theta = 0$$

Also from the triangle

$$ds\ sin\theta \;=\; dz, \quad \text{and} \quad ds\ cos\theta \;=\; dx$$

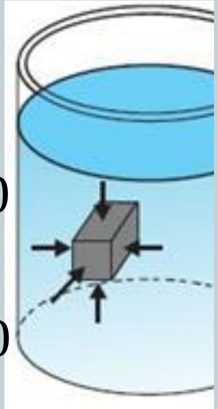
$$p_x \;=\; p_z \;=\; p_n \;=\; p$$

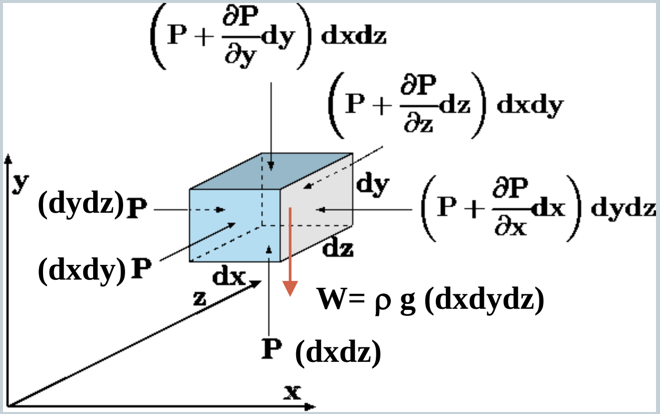
Static Pressure

Variation of pressure inside a static fluid

- Take a cuboid (dx, dy, and dz) in the static fluid as shown in Figure. The forces acting on the cuboid are also shown on each face.

Cuboid is at rest, so net force in each direction is zero





$$F_x = \mathrm{p(dydz)} - \left(P + \frac{\partial p}{\partial x}dx\right)\ dydz = 0$$

$$F_z = \mathrm{p(dxdy)} - \left(P + \frac{\partial p}{\partial z}dz\right)\ dxdy = 0$$

$$F_y = \mathrm{p(dxdz)} - \left(P + \frac{\partial p}{\partial y}dy\right)\ dxdz\ - \rho\ g\ (dxdydz) = 0$$

From above equations:

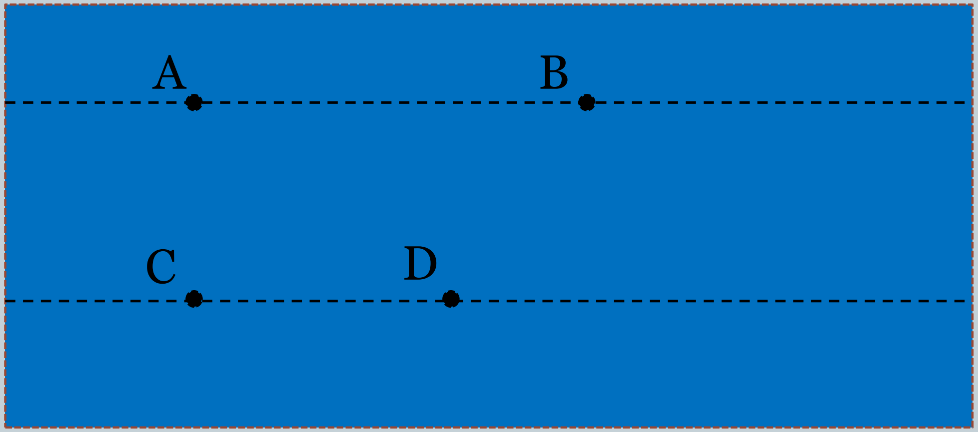
$$\frac{\partial p}{\partial x} = 0, \frac{\partial p}{\partial z} = 0, \frac{\partial p}{\partial y} = -\rho g dy$$

- This implies pressure in x and z directions (horizontal direction) is constant for a static fluid.
- However, the pressure in y direction (vertical direction) is variable.

Static Pressure

Variation of pressure inside a static fluid

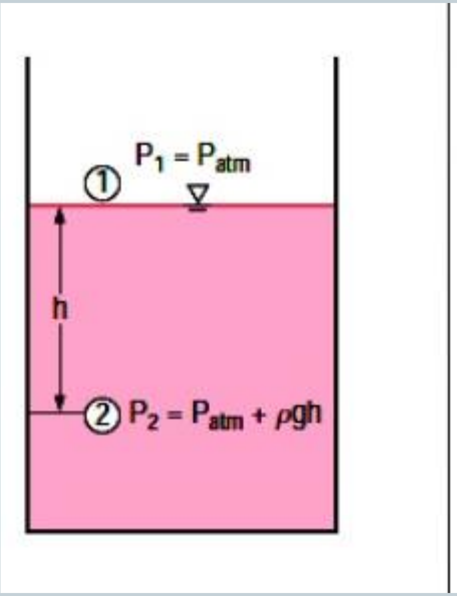
- If we will take two points A and B as shown in Figure.
- The pressure is same at A and B because they are same horizontal level.
- However pressure at C is different from A and B.
- Again, pressure at C and D are same because they are same horizontal level.
- Pressure at C is more than A, because C is at more depth than A.



Static Pressure

Variation of pressure with depth

- As discussed earlier, pressure vary in the vertical direction for static fluid.
- To analyze, if we take a point 1 to be at the free surface of a liquid open to the atmosphere, where the pressure is the atmospheric pressure P_{atm} then the pressure at a depth h from the free surface becomes by using equation givwn in the previous slide.



$$\frac{\partial p}{\partial y} = -\rho g dy, \qquad \int_2^1 \partial p = \int_0^h -\rho g dy \partial y$$

$$P_1 - P_2 = -\{(\rho g \text{h})\text{-}(\rho g \times 0)\} \qquad P_1 - P_2 = -\rho g \text{h}$$

$$P_2 - P_1 = \rho g \text{h} \qquad P_2 = P_1 + \rho g \text{h} \qquad P_2 = P_{atm} + \rho g \text{h}$$

Therefore pressure with the depth increases in a static fluid.

Static Pressure

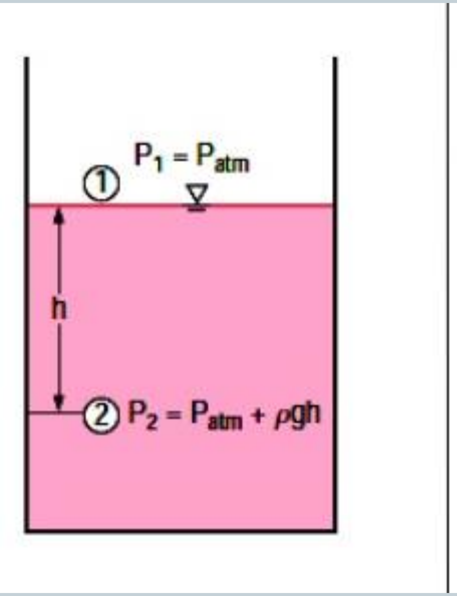
Pressure head

- Pressure head at point is the vertical height of that point from the free surface.
- From the equation given in the last slide

$$P_2 = P_{atm} + \rho g \mathrm{h} \qquad \rho g \mathrm{h} = P_2 - P_{atm}$$

$$\mathrm{h} = \frac{P_2 - P_{atm}}{\rho g}$$

Pressure head



Static Pressure

Fluid statics forces on submersed surface

Pressure Force

- When a stationary fluid comes in contact with solid surface either plane or curved, a force is exerted by the fluid on the surface. This force is called total pressure or pressure force or hydrostatic force. Since for a liquid at rest, no tangential force exists, the hydrostatic force acts in the direction normal to the surface. The point of application of total pressure on the surface is called center of pressure (CP).

Hydrostatic Force acting on Plane Surface

A plane surface may be immersed in a static liquid either in one position or combination of more than one position:

- A plane surface immersed horizontally
- A plane surface immersed Vertically
- A plane surface immersed at an angle i.e. Inclined position.

In every position of immersed plane surface, magnitude of total hydrostatic force and position of center of total hydrostatic force are to be determined.

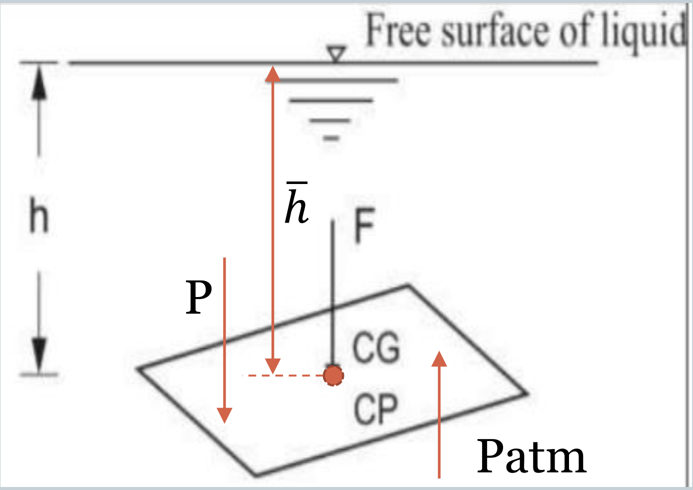
Static Pressure

Total Hydrostatic force (F) on Horizontal Plane Surface:

- Let a plane surface of area, A is immersed horizontally (Figure) in a stationary liquid of known specific weight, $\omega = \rho g$ and at a depth of h below free surface of the liquid. If P is the pressure intensity exerted by the liquid in stationary condition on the upper side of the plane. Further, the bottom of the plane is assumed to have atmospheric pressure.

$F = (P - P_{atm})A \qquad P = P_{atm} + \omega h$

$$F = \omega h A = \rho g h A$$



F = Total hydrostatic force acting on plane surface immersed horizontally in downward direction (N).

ρ = Density of liquid, (kg/m^3).

h = Depth of the plane surface below free surface of the liquid, (m) ($\bar{x}$).

A = Area of the plane surface (m^2).

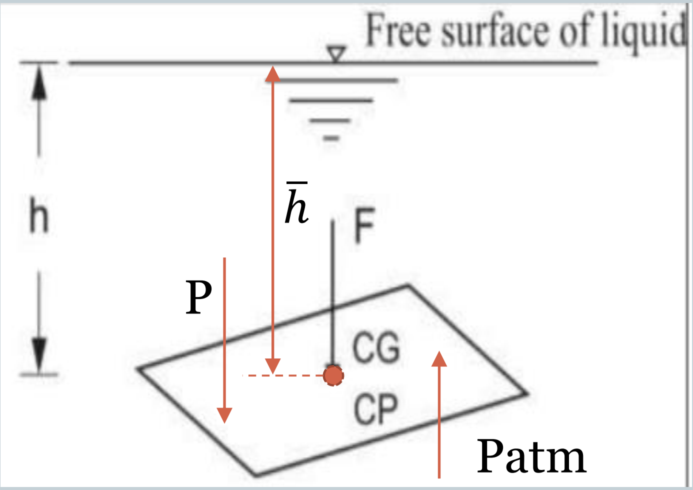
Static Pressure

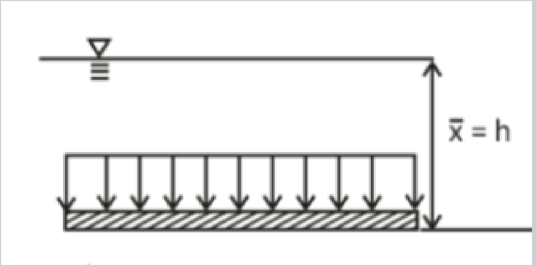
Total Hydrostatic force (F) on Horizontal Plane Surface:

Center of Pressure ($\bar{h}$) : Location of center of pressure (CP) is defined as the height of this point from free surface and denoted as $\bar{h}$.

For such condition, since pressure intensity is uniform over the area, the total hydrostatic force would pass through the centroid of the area (CG) and center of pressure (CP) coincide with each other.

$\bar{h} = h$





Static Pressure

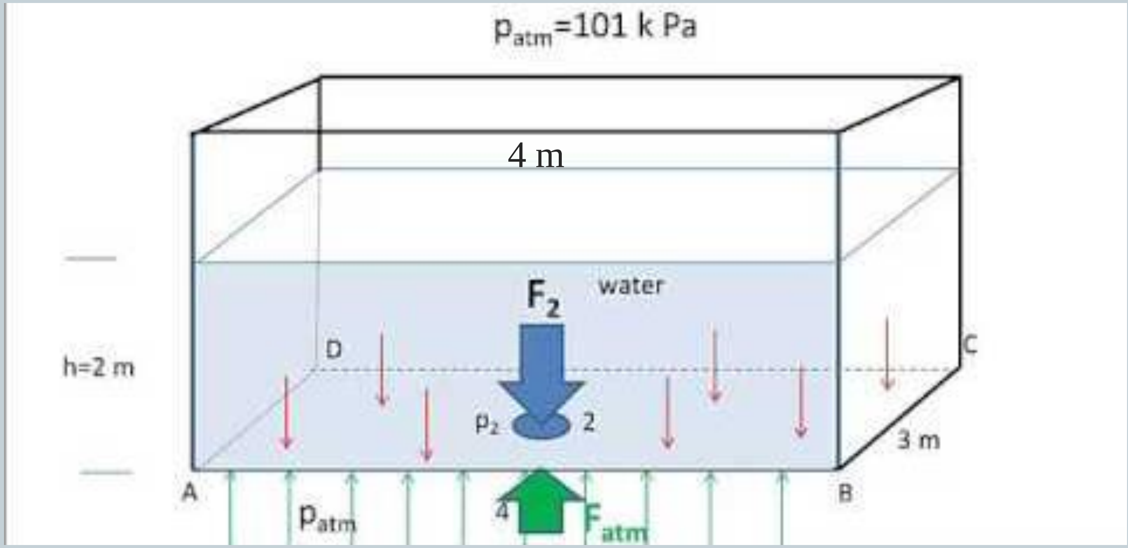
Solved Numerical: Find out the total pressure force on the bottom surface of the container as shown in Figure.

Solution:

$$p_2 = p_{atm} + \rho g h$$

$$F_2 = p_2 \times \text{Area}$$

$$F_{atm} = p_{atm} \times \text{Area}$$



$$F_{net} = F_2 - F_{atm} = \rho g h \times \text{Area} = 1000 \times 9.8 \times 2 \times (4 \times 3) = 235.2 \text{ kN}$$

$$\bar{h} = h = 2\text{m}$$

Static Pressure

Total Hydrostatic force (F) on Vertical Plane Surface:

- Let a plane surface of area, A is immersed vertically (Figure) in a stationary liquid of known specific weight, ω = ρg and at a depth of h (for centroid of the syrface) below free surface of the liquid. If P is the pressure intensity (variable) exerted by the liquid in stationary condition on the one side of the plane. Further, the other side of the plane is assumed to have atmospheric pressure. Then

$$F = \rho g \bar{x} A$$

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$$

F = Total hydrostatic force acting on plane surface immersed vertically (N).

x̄ = Depth of centroid of plane surface below free surface of the liquid, (m)

A = Area of the plane surface (m^2).

I_G = Moment of inertia of the surface about horizontal axis passes through centroid (G)

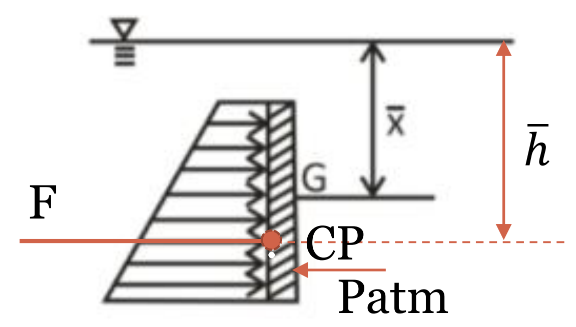
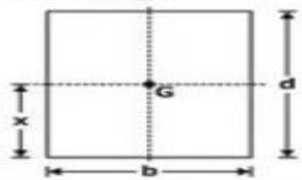
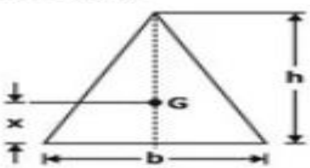
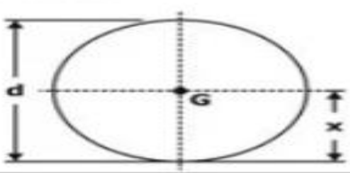
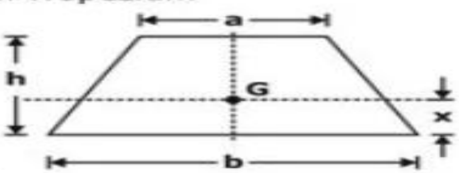


Fig: Vertical submerged surface

ρ = Density of liquid, (kg/m^3).

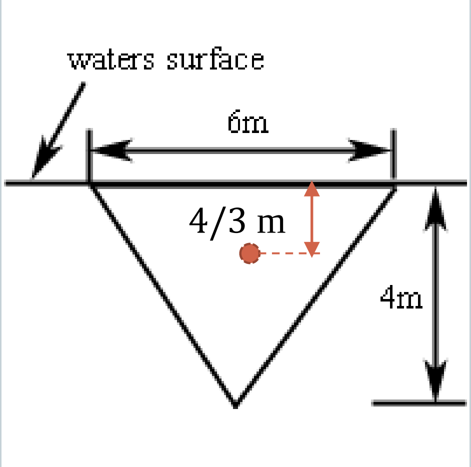
Static Pressure

Moment of Inertia formulas: Remember these

Plane surface	C.G. from the base	Area	Moment of inertia about an axis pasing through C.G. and parallel to base (I_G)
1. Rectangle 	$x = \frac{d}{2}$	bd	$\frac{db^3}{12}$
2. Triangle 	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$
3. Circle 	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$
4. Trapezium 	$x = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\frac{(a+b)}{2} \times h$	$\left(\frac{a^2+4ab+b^2}{36(a+b)} \times h^3\right)$

Static Pressure

Solved Numerical: Determine the hydrostatic force on the following triangular plate that is submerged in water as shown.



Solution:

$\bar{x}$ = 4/3 = 1.33 m A = $\frac{6\times4}{2}$ =12 m^2

F = $\rho g \bar{x}$A =1000×9.81×1.33×12 = 156960 N

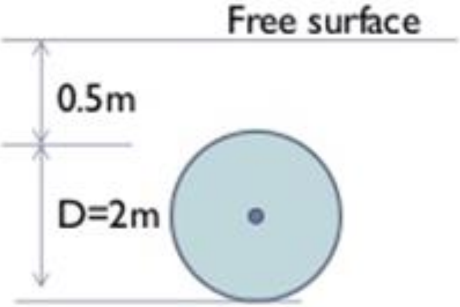
$$I_G = \frac{bh^3}{36} = \frac{6\times4^3}{36} = 10.667$$

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x} \qquad \bar{h} = \frac{10.667}{12\times1.33} + 1.33 = 2 \text{ m}$$

Static Pressure

Problem

- A plane surface is circular with a diameter of 2m. If it is vertical and the top edge is 0.5m below the water surface, find the magnitude of the force on one side and the depth of center of pressure.
- Solution:



$$h_c = 0.5 + \frac{D}{2} = 1.5m$$

$$F = \gamma h_c A$$

$$F = 9.810(1.5)\left(\frac{\pi}{4}2^2\right)$$

$$F = 46.2kN$$

$$h_p = h_c + \frac{I_c}{h_c A}$$

$$h_p = 1.5 + \left(\pi D^4 / 64\right) / \left(1.5 \times \pi D^2 / 4\right)$$

$$h_p = 1.667m$$

$$h_p = h_c + \frac{I_c}{h_c A}$$

$$F = \gamma h_c A$$

Chapter 3: Pressure and its Measurement

- Fluid Pressure: If the fluid is stationary , then the force exerted by the surrounding fluid on the area dA will be perpendicular to the surface dA. If dF is the force acting on dA in normal direction then intensity of pressure or pressure is given as

$$\boxed{\begin{array}{l} p = dF/dA \\ p = \text{Force/Area} \end{array}}$$

Unit: N/m2 known as Pascal (Pa)

- Pascal's Law: The pressure or intensity of pressure at a point in a static fluid is equal in all directions.
- Absolute pressure (P_{ab}): The pressure which is measured with reference to absolute vacuum pressure. In other words, it is measured above the absolute zero or complete vacuum.

Absolute Pressure = Atmospheric Pressure + Gauge pressure

$$P_{ab} = P_{atm} + P_{gauge}$$

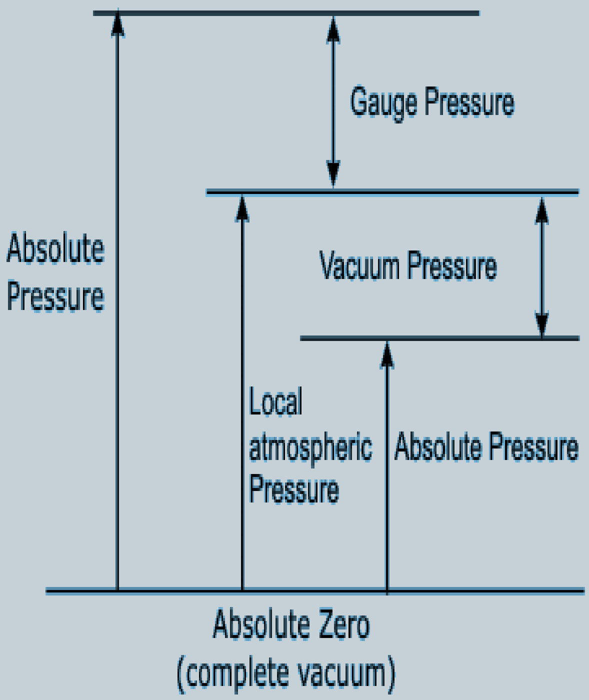
Pressure and its Measurement

- Gauge Pressure: It is the pressure which is measured above the atmospheric pressure, i.e. atmospheric pressure is taken as datum. It is measured with the help of a pressure measuring instrument.

- Vacuum Pressure: It is defined as the pressure below the atmospheric pressure.

Vacuum pressure = Atmospheric pressure – Absolute pressure

Example: What are the gauge pressure and absolute pressure at a point 3 m below the free surface of a liquid having a density of $1.53 X\ 10^3\ kg/m^3$ if the atmospheric pressure is equal to 750 mm /hg? The sp. Gravity of Hg is 13.6 and density of water is $1000\ kg/m^3$



Pressure and its Measurement

Solution : Depth of liquid Z1 = 3 m

Density of liquid ρ = 1.53 X 10^3 kg/m^3

Absolute pressure head Z_0 = 750 mm Hg

ρ_0 = Density of Hg = 13.6 X 1000 Kg/m^3

Atmospheric Pr., $p_{atm} = \rho_0$ X g X Z_0 = 13.6 X 1000 X 9.81 X .75

= 100062 M/m^2

Pressure at depth of 3m from free surface,

Gauge Pressure, P = ρ_1 X g X Z_1

= 1.53X1000X9.81X3 = 45028 N/m^2

Absolute Pressure, = Gauge Pressure + Atmospheric Pressure

= 45028 + 100062 = 145090 N/m^2

Pressure and its Measurement

The pressure of a fluid is measured by following devices :

1. Manometers
2. Mechanical Gauges

Manometers: theses are used for measuring pressure at a point in a fliud by balancing the column of fluid by the same or another column of fluid. They are classified as:

- Simple manometers
- Differential Manometers

Simple Manometers: It consists of a glass tube , having one of its ends connected to a point where pressure is to be measured and the other end remains open to atmosphere. Common types of simple manometers are:

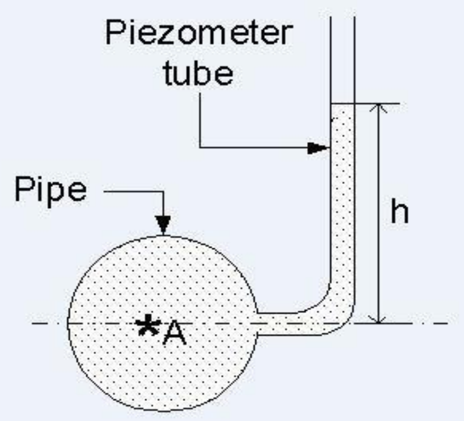
- Piezometer
- U-tube manometer
- Single column manometer

Pressure and its Measurement

Pressure Measurement

- Manometers
  - Simple Manometers
    - Piezometers
    - U-tube Manometer
    - Single Column Manometer
    - Inclined Single Column Manometer
  - Differential Manometers
    - Double Piezometers
    - U-tube Differential Manometer
    - Inverted U-tube Differential Manometer
    - Micromanometer
- Mechanical gauges
  - Diaphragm Pr. Gauge
  - Bourdon Tube
  - Dead weight Pr. Gauge
  - Bellows Pr. Gauge

Pressure and its Measurement



Piezometer: Simplest form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and the other end is open.

pressure at A = ρ x g x h N/m^2

ρ is density of liquid in piezometer.

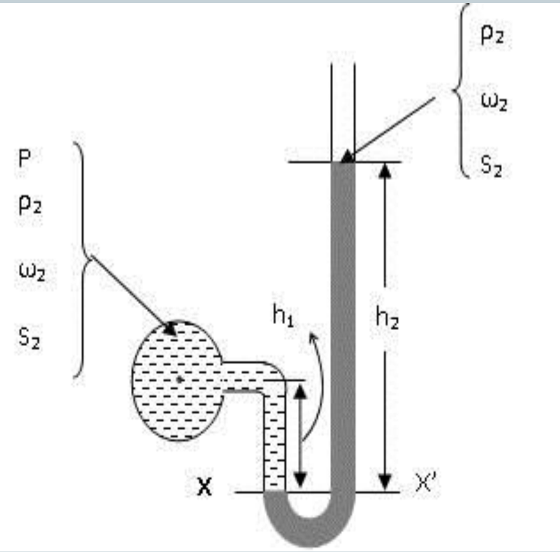
g is acceleration of gravity.

h is height of liquid column(as shown in figure).

Pressure and its Measurement

U-Tube Manometer: It consists of a glass tube bent in U shape, one end of which is connected to a point at which pressure is to be measured and other is open to atmosphere.

- The tube generally contains mercury or any other liquid whose sp. Gravity is greater than sp. Gravity of the liquid whose pressure is to be measured.



Pressure at point B will be given as:

$$p = \rho_2 g h_2 - \rho_1 g h_1 \text{ (symbols have the usual meaning)}$$

Example: The right limb of a simple u-tube manometer, containing Hg, is open to the atmosphere while the left limb is connected to a pipe in which fluid of sp. Gravity 0.9 is flowing. The centre of pipe is 12 cm below the level of Hg in the right limb. Find the pressure of fluid in the pipe if the difference of Hg level in the two limbs is 20 cm.

Pressure and its Measurement

Solution: $S_1 = 0.9$

$\rho_1 = S_1 \times 1000 = 900$ Kg/m^3

$S_2 = 13.6$

$\rho_2 = S_2 \times 1000 = 13600$ Kg/m^3

h = 12 cm

h_2 = 20 cm = .2 m

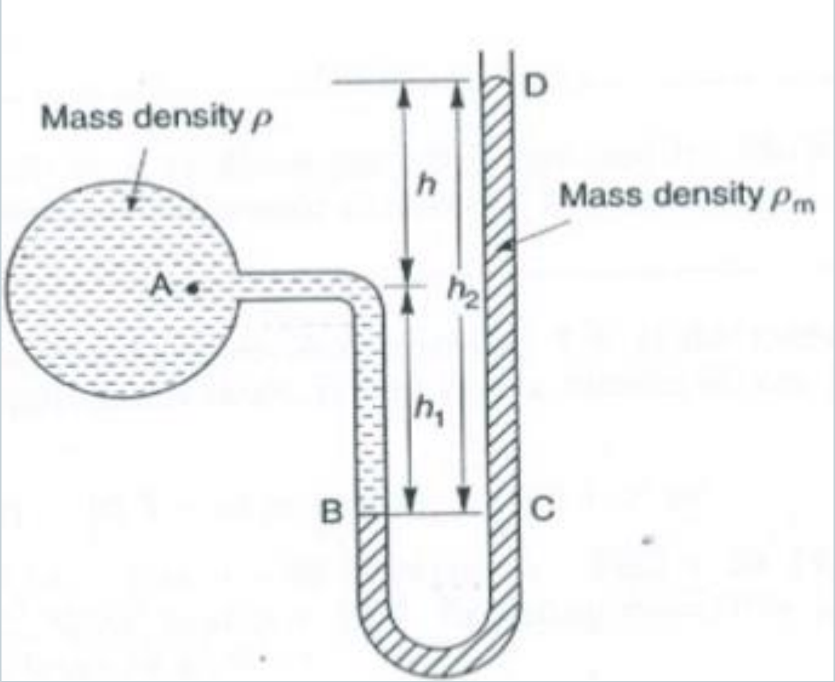
h_1 = 20- 12 = 8 cm = 0.08 m

$P + \rho_1 g h_1 = \rho_2 g\, h_2$

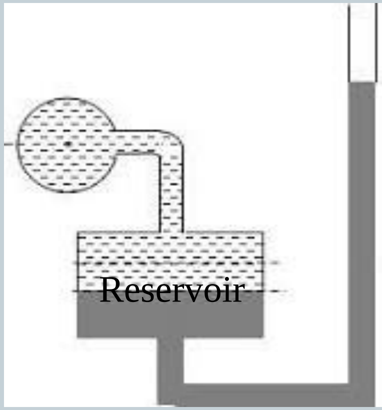
P = 13600 x 9.81 x .2 – (900 x 9.81 x 0.08)

= 26683 – 706

Pressure at A = 25977 N/m^2



Pressure and its Measurement

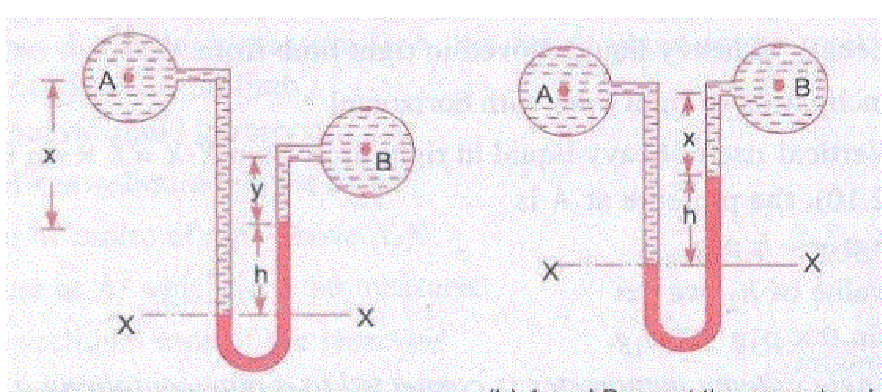


Single column Manometer: It is a modified form of U-tube manometer, in which a reservoir , having a large cross-sectional area as compared to the area of tube is connected to one of the limbs of the manometer.

Differential Manometer: These devices are used for measuring the difference of Pressure between two points in a pipe or in two different pipes.

U Tube Differential Manometer:

It consists of a u-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference



(a)Two pipes at different levels (b) A and B are at the same level

Pressure and its Measurement

- ρ_1 is density of liquid at A; ρ_2 is density of liquid at B and ρ_g is density of heavy liquid or mercury
- P_a is pressure at point A and P_b is pressure at point A
- In the given figure (a), pressure in the left limb is = $\rho_1 g (h + x) + P_a$

Pressure in right limb = $\rho_g gh + \rho_2 gy + P_b$

Equating the two pressures, we get,

$\rho_1 g (h + x) + P_a = \rho_g gh + \rho_2 gy + P_b$

$P_a - P_b = \rho_g gh + \rho_2 gy - \rho_1 g (h + x)$

$= hg (\rho_g - \rho_1) + \rho_2 gy - \rho_1 g x$

In the given figure (b), pressure in the left limb is = $\rho_1 g (h + x) + P_a$

Pressure in right limb = $\rho_g gh + \rho_1 gx + P_b$

Pressure and its Measurement

Equating the two pressures, we get,

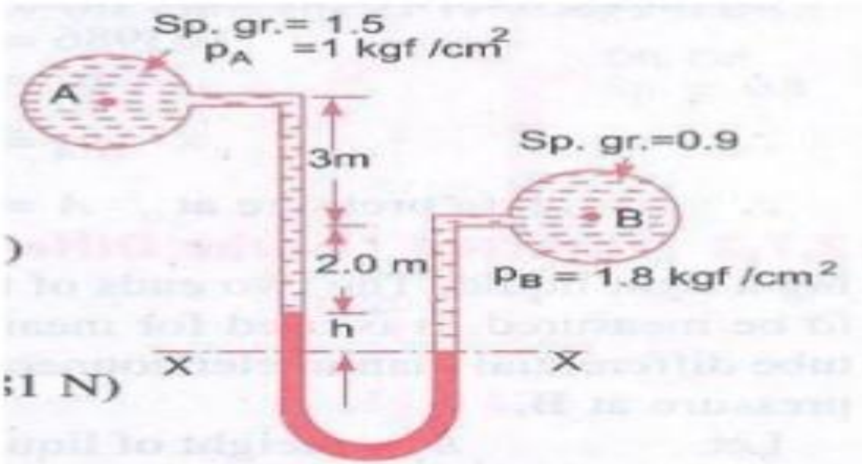
$$\rho_g gh + \rho_1 gx + P_b = \rho_1 g\,(h + x) + P_a$$

$$P_a - P_b = \rho_g gh + \rho_1 gx - \rho_1 g\,(h + x)$$

$$= hg\,(\rho_g - \rho_1)$$

- Taking an example,

A differential manometer is connected at the two points A and B of two pipes as shown in Fig. The pipe A contains a liquid of sp. gr. = 1,5 while pipe B contains a liquid of sp. gr 0.9. The pressures at A and B are 1 kgf/cm² and 1.80 kgf/cm² respectively. Find the difference in mercury level in the differential manometer. (1 kgf=9.81 N)



Pressure and its Measurement

Solution: At A $S_1 = 1.5$ $\rho_1 = 1500$ Kg/m^3

At B $S_2 = 0.9$ $\rho_2 = 900$ Kg/m^3

$P_a = 1$ kgf/cm^2 = $1\text{x}10^4$ kgf/m^2

= 10^4 x 9.81 N/m^2

$P_b = 1.8$ kgf/cm^2 = 1.8 x 10^4 x 9.81 N/ m^2

Density of Hg = 13.6 x 1000 Kg/m^3

Taking X-X as datum line,

Pressure in left limb above x-x = 13.6 x 1000 x 9081 x h + (1500 x 9.81 x 5) + P_a

= 13.6 x 1000 x 9.81 x h + (1500 x 9.81 x 5) + 10^4 x 9.81 N/m^2

Pressure and its Measurement

Pressure in right limb above x-x = $900 \times 9.81 \times (h+2) + P_b$

$= 900 \times 9.81 \times (h+2) + 1.8 \times 10^4 \times 9.81 \text{ N/ m}^2$

Equating the two pressures, we get,

$$13.6 \times 1000 \times 9.81 \times h + (1500 \times 9.81 \times 5) + 10^4 \times 9.81 = 900 \times 9.81 \times (h+2) + 1.8 \times 10^4 \times 9.81$$

$$13.6\,h + 7.5 + 10 = (h + 2) \times .9 + 18$$

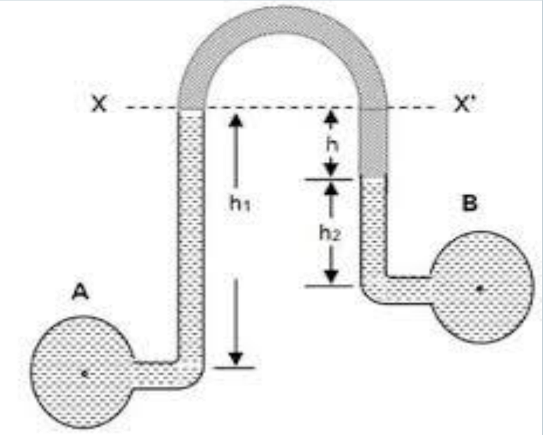
$$13.6h + 17.5 = 0.9h + 1.8 + 18 = 0.9h + 19.8$$

$$12.7h = 2.3$$

$$h = 0.181\text{m Ans}$$

Pressure and its Measurement

Inverted U-Tube Differential Manometer: It consists of a inverted U- tube, containing a light liquid. The two point are connected to points whose pressure difference is to be measured.



- ρ_1 is density of liquid at A; ρ_2 is density of liquid at B and ρs is density of light liquid.
- P_a is pressure at point A and P_b is pressure at point A
- Taking X-X as datum line,
- In the given figure, pressure in the left limb below x-x is = $P_a - \rho_1 g h_1$

  pressure in the right limb below x-x is = $P_b - \rho_2 g h_2 - \rho_s g h$

Pressure and its Measurement

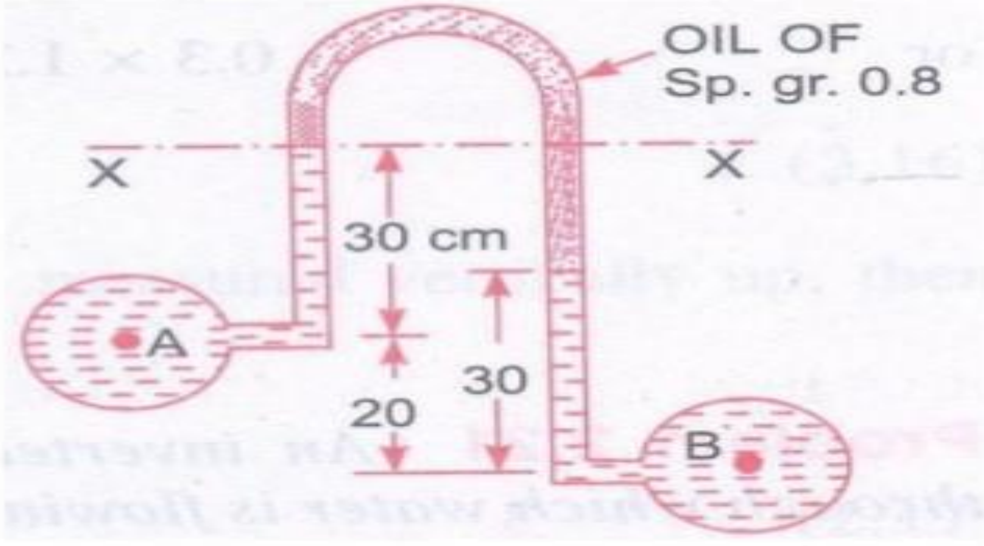
Equating the two pressures, we get,

$$P_a - \rho_1 g h_1 = P_b - \rho_2 g h_2 - \rho_s g h$$

$$P_a - P_b = \rho_1 g h_1 - \rho_2 g h_2 - \rho_s g h$$

Taking an example,

In Fig. an inverted differential manometer is connected to two pipes A and B which convey water. The fluid in manometer is oil of sp. gr. 0.8. For the manometer readings shown in the figure, find the pressure difference between A and B.



Pressure and its Measurement

Solution: Sp. Gr. Of oil = 0.8 i.e. $\rho_s = 800$ Kg/m^3

Difference of oil in two limbs = (30+20) – 30 = 20cm

Taking X-X as datum line,

Pressure in left limb below x-x = $P_a - 1000 \times 9.81 \times 30$

$= P_a - 2943$

Pressure in right limb below x-x = $P_b - 1000 \times 9.81 \times 0.3 - 800 \times 9.81 \times 0.2$

$= P_b - 2943 - 1569.6$

$= P_b - 4512.6$

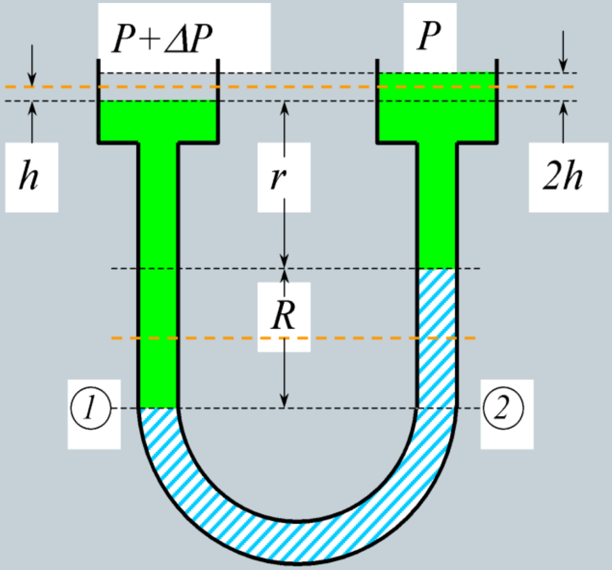
Equating the two pressures, we get,

$P_a - 2943 = P_b - 4512.6$

$P_a - P_b = 4512.6 - 2943 = 1569.6$ N/m^2

Pressure and its Measurement

- Micromanometer: It is a combination of two single column manometers. This is used for measurement of very small pressure differentials or where the pressure differential is to be measured with very high accuracy.



Mechanical Gauge

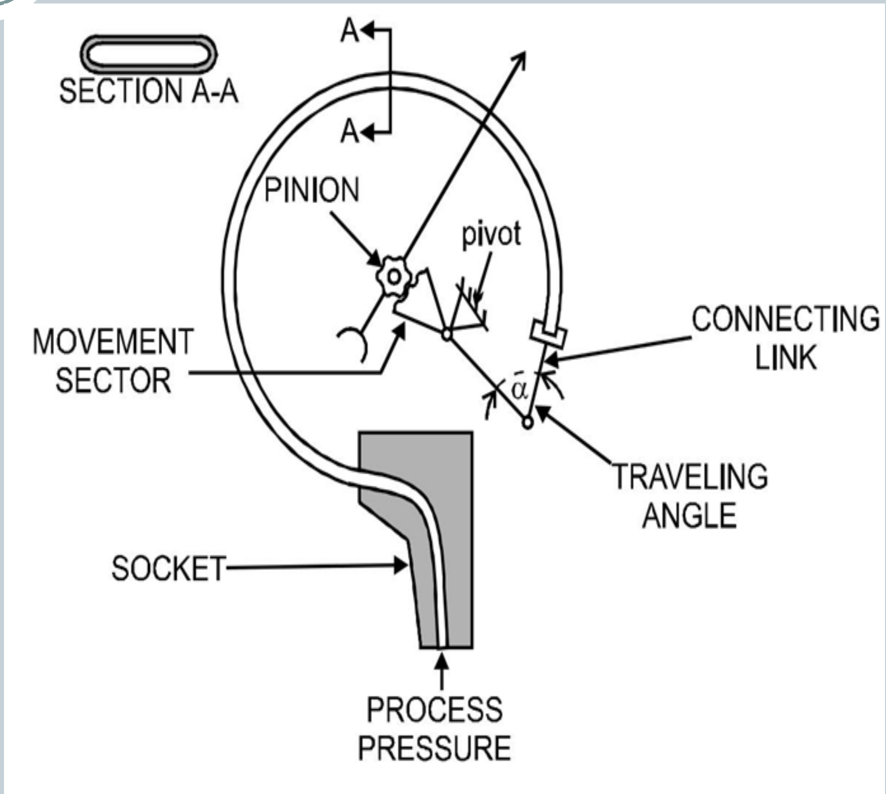
Bourdon Tube: The Bourdon tube is the namesake of Eugéne Bourdon, a French watchmaker and engineer who invented the Bourdon gauge in 1849.

- Bourdon tube pressure gauges are very common and are used to measure medium to high pressures. They cover measuring spans from 600 mbar to 4,000 bar.

Pressure and its Measurement

- Bourdon tube is an elastic-element type of pressure transducer. It is relatively cheap and is commonly used for measuring the gauge pressure of both gaseous and liquid fluids.

- The Bourdon pressure gauge operates on the principle that, when pressurized, a flattened tube tends to straighten or regain its circular form in cross-section.



C-Bourdon pressure element

- The Bourdon tube comes in C, helical, and spiral shapes.

Chapter 4: Flow of Fluids

Types of Flow:

A fluid may be in static condition or in motion. For a fluid in static condition, specific weight of the fluid plays an important role. But when the fluid is in motion, various other fluid properties (velocity, density, pressure, temperature etc.) become significant. The fluid flow is classified as:

- Steady and Unsteady flows
- Uniform and Non-uniform flows
- Laminar and Turbulent flows
- Rotational and Irrotational flows
- Compressible and Incompressible flows

Flow of Fluids

Steady and Unsteady flows

- **Steady flow:** It is defined as that type of flow in which the fluid properties like velocity, pressure, density etc. at a point do not change with time.

  Mathematically, for steady flow

  $$\frac{\partial v}{\partial t} = 0 \qquad \frac{\partial p}{\partial t} = 0 \qquad \frac{\partial \boldsymbol{\rho}}{\partial t} = 0$$

  Example: Flow of an incompressible fluid (i.e. liquids) through a pipeline

- **Unsteady flow:** It is defined as that type of flow in which the fluid properties like velocity, pressure, density etc. at a point change with time.

  Mathematically, for steady flow

  $$\frac{\partial v}{\partial t} \neq 0 \qquad \frac{\partial p}{\partial t} \neq 0 \qquad \frac{\partial \boldsymbol{\rho}}{\partial t} \neq 0$$

  Example: Flow of compressible fluid (i.e. gas) through a pipeline

  The flow in a pipe whose valve is being opened or closed gradually.

Flow of Fluids

Uniform and Non-Uniform flows

- **Uniform flow:** It is defined as that type of flow in which the velocity of flow of a fluid is constant at any section in the path of flow of the fluid. Mathematically, for uniform flow

$$\left(\frac{\partial v}{\partial s}\right)_{t=constant} = 0$$

where ∂v=change of velocity

∂s= Displacement in any direction

Example: Flow of a liquid through a pipeline of uniform diameter.

- **Non-Uniform flow:** It is defined as that type of flow in which the velocity of flow of a fluid is different at different sections in the path of flow of the fluid. Mathematically, for Non- uniform flow

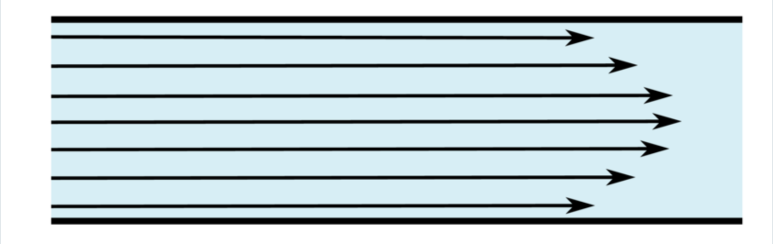
$$\left(\frac{\partial v}{\partial s}\right)_{t=constant} \neq 0$$

Example: Flow of a liquid through a pipeline of variable diameter.

Flow of Fluids

Laminar and Turbulent flows

- **Laminar flow:** A flow is said to be laminar if each particle fluid has a definite path and the path of one particle does not cross the path of any other particle.



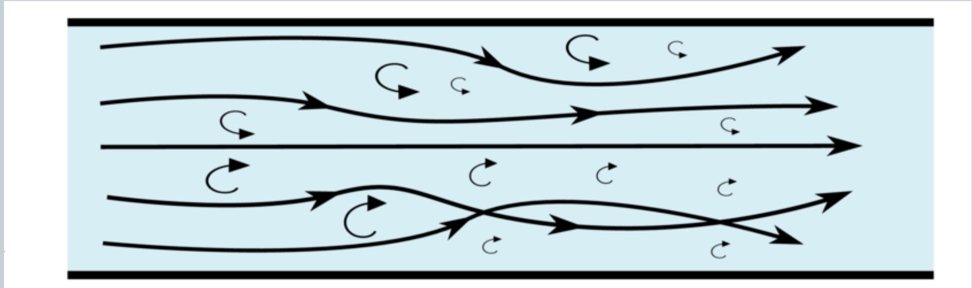
Laminar flow

Laminar flow is also called streamline or viscous flow. Such a flow can occur only when the velocity of flow is low.

Example: Ground water flows, Flow of blood in veins and arteries, flow of muddy water at a very low velocity through a pipeline, flow through a capillary tube.

Flow of Fluids

- **Turbulent flow**: A flow is said to be turbulent if the fluid particle do not have a definite path and the path of one particle crosses the path of other particles during flow.



Turbulent flow

In turbulent flow the fluid particles move in a zig-zag way. Turbulent flow is also called Non-laminar flow. This type of flow occur when the velocity of flow is high.

Example: Flow of a liquid of low viscosity such as petrol through a pipeline

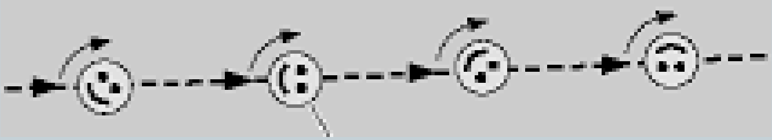
Flow of Fluids

Compressible and Incompressible flows:

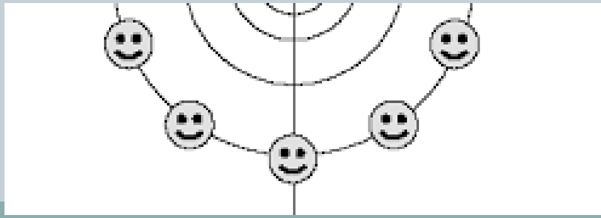
- **Compressible flow:** is the branch of fluid mechanics that deals with flows having significant changes in fluid density is called compressible flow.
- **Incompressible flow:** is the branch of fluid mechanics that deals with flows having significant no changes in fluid density is called incompressible flow.

Rotational and Irrotational flows:

- **Rotational flows:** Fluid particles rotate about their mass centre while moving in the direction of flow. This flow is called rotational flow. Example: Forced vortex, earth



- **Irrotational flows:** The fluid particles do not rotate about their mass centre while moving in the direction of flow. This flow is called irrotational flow. Example: Free vortex, satellite, whirlpool in a river.



Flow of Fluids

Discharge (Rate of Flow):

It is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel. It is generally denoted as Q.

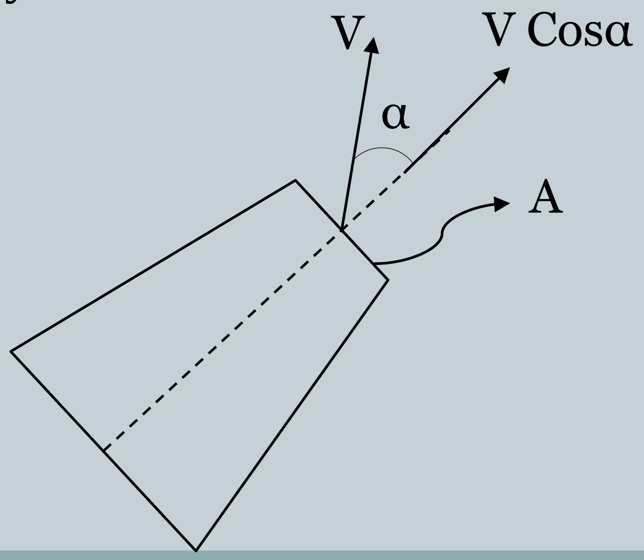
Consider a liquid flowing through a pipe.

Let A=Cross sectional area of the pipe

V= Average velocity of the liquid in perpendicular direction of cross-section

Discharge, Q =Area × Average velocity

$$Q = AV\cos\alpha \ (m^3/sec)$$



Flow of Fluids

CONTINUITY EQUATION:

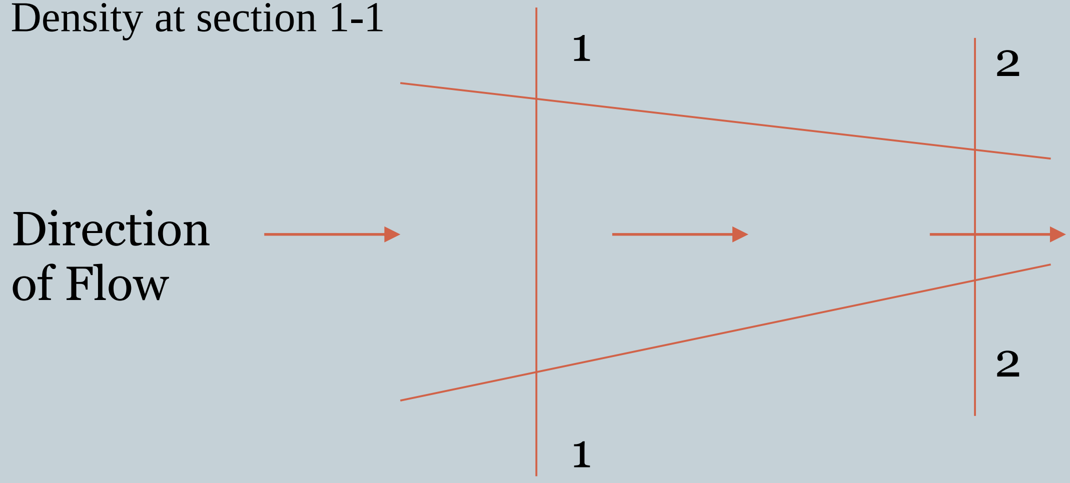
It states that if no fluid is added or removed from the pipe in any length then the mass passing across different sections shall be same. Continuity equation is based on the principle of conservation of mass.

Consider two cross sections of a tapering pipe as shown in diagram.

Let $\mathbf{V}_1$= Average velocity at section 1-1

$\mathbf{A}_1$= Area of pipe at section 1-1

$\boldsymbol{\rho}_1$= Density at section 1-1



Flow of Fluids

CONTINUITY EQUATION:

and $\mathbf{V_2}$, $\mathbf{A_2}$, $\boldsymbol{\rho}_2$ are corresponding values at section 2-2.

Total quantity of fluid (mass) passing through section 1-1= $\boldsymbol{\rho_1 A_1 V_1}$

Total quantity of fluid (mass) passing through section 2-2= $\boldsymbol{\rho_2 A_2 V_2}$

From the law of conservation of mass, we have

$$\boldsymbol{\rho_1 A_1 V_1 = \rho_2 A_2 V_2} \qquad \textbf{---(1)}$$

Equation (1) is called Continuity equation. It is applicable to the compressible (i.e. gases) as well as incompressible (i.e. liquids) fluids.

For incompressible fluids $\boldsymbol{\rho}_1 = \boldsymbol{\rho}_2$

So, from continuity equation (1), we get

$$\mathbf{A_1 V_1 = A_2 V_2}$$

Flow of Fluids

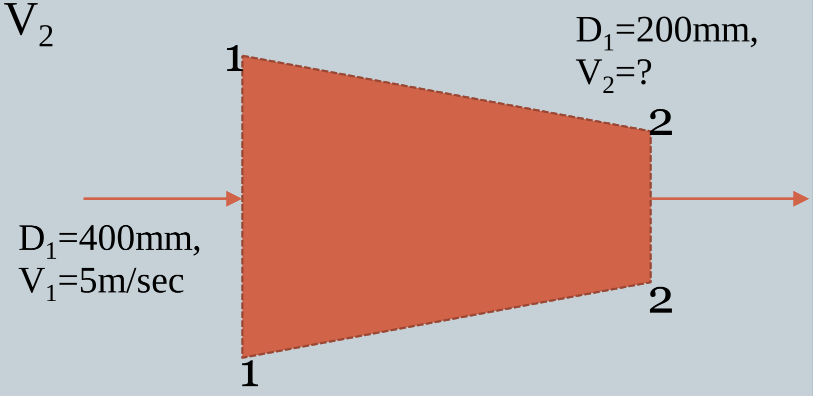
CONTINUITY EQUATION:

1. The diameter of a pipe at the sections 1-1 and 2-2 are 400 mm and 200 mm respectively. If the velocity of water flowing through the pipe at section 1-1 is 5m/sec, find:
   a) Discharge through the pipe.
   b) Velocity of water at section 2-2

Solution: $D_1 = 400\text{mm}$, $D_2 = 200\text{mm}$, $V_1 = 5\text{m/sec}$,

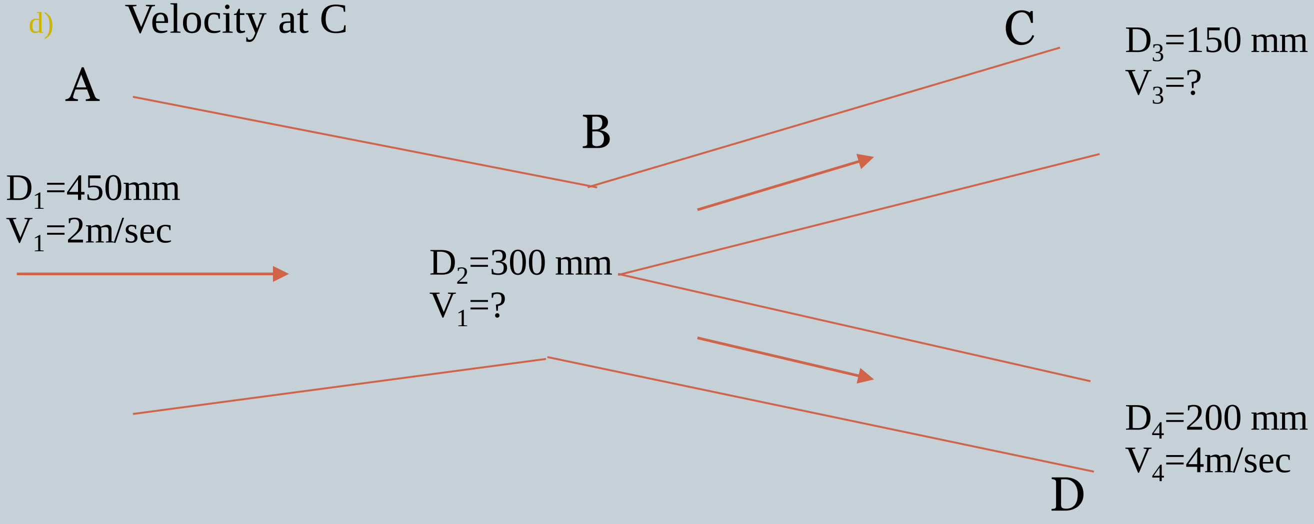
Discharge = $A_1 V_1 = \frac{\pi}{4} D_1^2 \times 5 = 0.6284\ \text{m}^3\text{/sec.} = A_2\ V_2$

$V_2 = A_2 / 0.6284 = .03142/.6284 = 0.05\ \text{m/sec}$



Flow of Fluids

1. Oil flows through a pipeline which contracts from 450mm diameter at A to 300mm diameter at B and the branches into two pipes C and D. the diameter of the pipe C is 150mm and the diameter of the pipe D is 200mm. If the velocity at A is 2m/sec and that at D is 4m/sec, Determine
   a) Velocity at section B
   b) Discharge at D
   c) Discharge at C
   d) Velocity at C



Flow of Fluids

- Solution: At A: Diameter D_1=450 mm=0.45 m

  Area $A_1=0.159 m^2$

  V_1 = 2m/sec
- At B: Diameter D_2=300 mm=0.30 m

  Area $A_2=0.0707 m^2$
- At C: Diameter D_3=150 mm=0.15 m

  Area $A_3=0.0177\ m^2$
- At D: Diameter D_4=200 mm=0.20 m

  Area $A_4=0.0314 m^2$

  V_4 = 4m/sec

Apply Continuity equation:

$$A_1V_1=A_2V_2$$

Flow of Fluids

V_2=(0.159X2)/0.0707=4.5m/sec.

Discharge through pipe D, $Q_4 = A_4V_4$

$= 0.0314X\ 4=0.1256 m^3/sec$

Let Q_3= Discharge through pipe C

$Q_3 + Q_4 = Q_2$

$Q_3 + 0.1256 = A_2V_2$

$Q_3 + 0.1256 = 0.0707\ X\ 4.5=0.3182$

$Q_3 = 0.1926\ m^3/sec.$

$Q_3 = A_3\ V_3$

$V_3 = 0.1926/0.0177 = 10.9\ m/sec.$

Flow of Fluids

ENERGY OF AN IDEAL FLUID:

There are three types of energies or heads of flowing fluids:

1. **Potential energy**
2. **Kinetic energy**
3. **Pressure energy**

Potential energy: The energy possessed by a fluid particle by virtue of its position is called potential energy, Potential energy is due to position of fluid above some suitable datum line.

Potential head is the potential energy of a fluid per unit of its weight. Potential head is also called datum head or static head. It is denoted as Z.

Let m= Mass of a fluid particle in Kg

Z= Height of the fluid particle above the datum line in metre.

Weight of the fluid particle=mg

Potential energy of the fluid particle in Nm=mgZ

Potential head of the fluid particle=$\frac{mgZ}{mg}$ =Z metre of the fluid

Flow of Fluids

ENERGY OF AN IDEAL FLUID:

Kinetic Energy: The energy possessed by a fluid particle by virtue of its motion is called kinetic energy.

kinetic head or velocity head is the kinetic energy of a fluid per unit of its weight.

Let m= Mass of a fluid particle in Kg

V=velocity of flow of the fluid in m/sec.

Weight of the fluid particle=mg

Kinetic energy of the fluid particle in Nm=$\frac{1}{2}mv^2$

Kinetic head or velocity head of the fluid particle= $\frac{1}{2}mv^2/mg=\frac{V^2}{2g}$ meter of fluid

Flow of Fluids

ENERGY OF AN IDEAL FLUID:

Pressure Energy: The energy possessed by a fluid particle by virtue of its existing pressure is called Pressure energy.

Pressure head is the pressure energy of a fluid per unit of its weight.

Let W= Weight of the fluid particle

w= Specific volume of the fluid particle

Weight of the fluid particle=mg

Pressure energy of the fluid particle in Nm (Work done) = P × Volume = P $\times \frac{W}{w}$

Pressure head fluid particle= $\frac{P \times \frac{W}{w}}{W}$ = h= Depth of the horizontal pipeline below the free surface of the liquid.

Flow of Fluids

BERNOULLI'S THEOREM:

This theorem was given by Dr. Daniell Bernoulli's in 1783. He was a Swiss engineer. Bernoulli's theorem is a form of the principle of conservation of energy.

Bernoulli's theorem states that for an incompressible fluid flowing from one section to another in a continuous stream, the total energy of the fluid particle remain constant.

Mathematically,

$$\frac{P}{w} + \frac{V^2}{2g} + Z = \text{Constant}$$

Where $\frac{P}{w}$ = Pressure head

$\frac{V^2}{2g}$ = velocity head or kinetic head

Z=Potential head

Flow of Fluids

BERNOULLI'S THEOREM:

Assumptions:

1. The flow is ideal and incompressible
2. The flow is steady and continuous
3. The flow is one –dimensional i.e. along a stream line
4. The velocity is uniform over the section and is equal to the mean velocity.
5. No external force except force of gravity and pressure acts on the fluid.

LIMITATIONS OF BERNOULLI'S THEOREM :

1. Mean velocity of flow should be taken into consideration
2. Loss of energy due to pipe friction during flow of the liquid from one section to another section is neglected .
3. Bernoulli's equation does not take into consideration loss of energy due to turbulent flow.
4. Bernoulli's equation does not take into consideration loss of energy due to change of direction.

Flow of Fluids

Problem 1 *Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm² (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5 m above the datum line.*

Solution. Given :

Diameter of pipe $= 5 \text{ cm} = 0.5 \text{ m}$

Pressure, $p = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$

Velocity, $v = 2.0 \text{ m/s}$

Datum head, $z = 5 \text{ m}$

Total head = pressure head + kinetic head + datum head

Pressure head $= \dfrac{p}{\rho g} = \dfrac{29.43 \times 10^4}{1000 \times 9.81} = 30 \text{ m}$ $\left\{ \rho \text{ for water} = 1000 \dfrac{\text{kg}}{\text{m}^3} \right\}$

Kinetic head $= \dfrac{v^2}{2g} = \dfrac{2 \times 2}{2 \times 9.81} = 0.204 \text{ m}$

$\therefore$ Total head $= \dfrac{p}{\rho g} + \dfrac{v^2}{2g} + z = 30 + 0.204 + 5 =$ **35.204 m. Ans.**

Flow of Fluids

Problem 2 *Water is flowing through a pipe having diameter 300 mm and 200 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525 N/cm² and the pressure at the upper end is 9.81 N/cm². Determine the difference in datum head if the rate of flow through pipe is 40 lit/s.*

Solution. Given :

Section 1, $D_1 = 300 \text{ mm} = 0.3 \text{ m}$

$p_1 = 24.525 \text{ N/cm}^2 = 24.525 \times 10^4 \text{ N/m}^2$

Section 2, $D_2 = 200 \text{ mm} = 0.2 \text{ m}$

$p_2 = 9.81 \text{ N/cm}^2 = 9.81 \times 10^4 \text{ N/m}^2$

Rate of flow $= 40$ lit/s

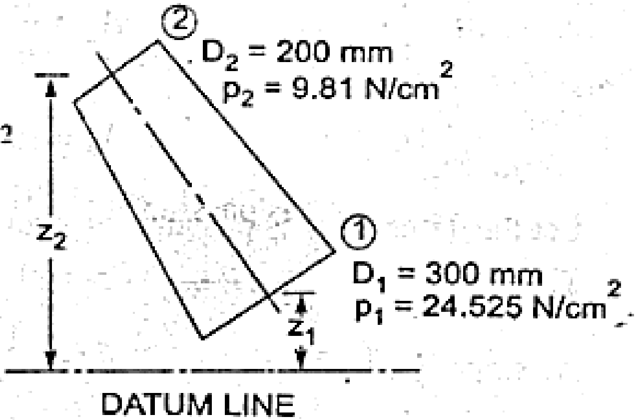


Fig. 6.4

or $$Q = \frac{40}{1000} = 0.04 \text{ m}^3/\text{s}$$

Now $$A_1V_1 = A_2V_2 = \text{rate of flow} = 0.04$$

$$\therefore \quad V_1 = \frac{.04}{A_1} = \frac{.04}{\frac{\pi}{4}D_1^2} = \frac{0.04}{\frac{\pi}{4}(0.3)^2} = 0.5658 \text{ m/s}$$

$$\simeq 0.566 \text{ m/s}$$

$$V_2 = \frac{.04}{A_2} = \frac{.04}{\frac{\pi}{4}(D_2)^2} = \frac{0.04}{\frac{\pi}{4}(0.2)^2} = 1.274 \text{ m/s}$$

Flow of Fluids

Applying Bernoulli's equation at (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{24.525 \times 10^4}{1000 \times 9.81} + \frac{.566 \times .566}{2 \times 9.81} + z_1 = \frac{9.81 \times 10^4}{1000 \times 9.81} + \frac{(1.274)^2}{2 \times 9.81} + z_2$$

or $$25 + .32 + z_1 = 10 + 1.623 + z_2$$

or $$25.32 + z_1 = 11.623 + z_2$$

$\therefore$ $$z_2 - z_1 = 25.32 - 11.623 = 13.697 = 13.70 \text{ m}$$

$\therefore$ Difference in datum head $= z_2 - z_1 = \mathbf{13.70}$ **m. Ans.**

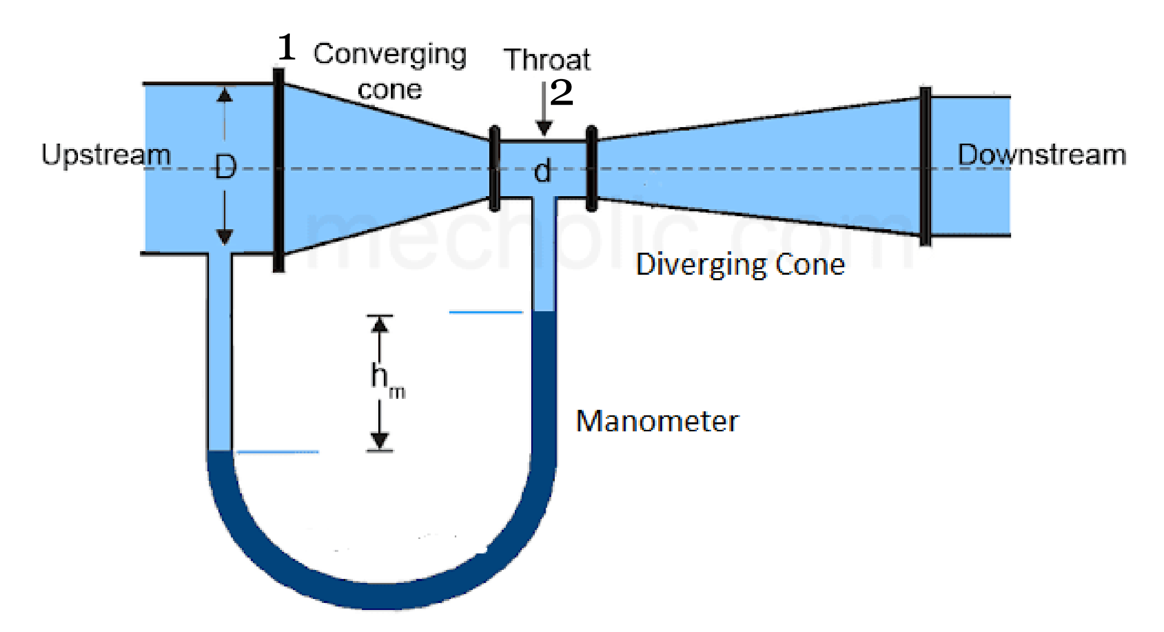
Flow of Fluids

- Applications of Bernoulli's Theorem used for flow measuring devices like
  a) Venturimeter
  b) Orifice meter
  c) Pitot Tube

Flow of Fluids

VENTURIMETER:

The principle of venturimeter is that when a fluid flows through the venturimeter, it accelerates in the convergent section and decelerates in the divergent section, resulting in a drop in the static pressure followed by a pressure recovery in the flow direction.



Flow of Fluids

VENTURIMETER:

The principle of venture meter is firstly developed by G.B. Venturi in 1797 but this principle comes into consideration with the help of C. Herschel in 1887. Its basic principle also depend on Bernoulli’s equation and continuity equation. Velocity increases pressure decreases.

Simple meaning is “When cross sectional area of the flow is reduces it creates pressure difference between the different areas of flow. This difference in pressure is measured with the help of manometer and helps in determining rate of fluid flow or other discharge from the pipeline.”

Main parts of Venturimeter:-

1. Converging part
2. Throat
3. Diverging Part

Flow of Fluids

Converging Part:

It is starting section of venturimeter which attached at inlet pipe. The cross-sectional area of this cone starts to decrease, and the converging angle is 20 degree. Its length is 2.7(D-d). Here (D) is the diameter of inlet section and (d) is the diameter of throat. Other end of converging is attached with throat.

Throat: Throat is middle portion of venturimeter, and its cross-sectional area is too small. At this point pressure is decreases and velocity is increases. One end relates to converging part and other end is attached with diverging part. Diameter of throat is ¼ to ¾ of the diameter of the inlet pipe, but mostly it is ½ of the diameter of the pipe.

Diverging part: Diverging part is last part of venturimeter and its cross-sectional area is increases continually. Angle of diverging part is 5 to 15 degree. Its cross-sectional area continuously increases. One end is connected to throat and other end is connected to outlet pipe. The main reason behind the low diverging angle is to avoid the formation of eddies because flow separation and eddies formation will results in large amount of loss in energy.

Flow of Fluids

Expression for the rate of flow through venturimeter:-

Let d_1, p_1, v_1 & a_1, are the diameter at the inlet, pressure at the inlet, velocity at the inlet and area at the cross section 1.

And d_2, p_2, v_2 and a_2 are the corresponding values at section 2.

Applying Bernoulli's equation at sections 1 and 2

$$\frac{P_1}{\rho g} + \frac{{V_1}^2}{2g} + \text{Z} = \frac{P_2}{\rho g} + \frac{{V_2}^2}{2g} + \text{Z}$$

As the pipe is horizontal, so $Z_1 = Z_2$

$$\frac{P_1}{\rho g} + \frac{{V_1}^2}{2g} = \frac{P_2}{\rho g} + \frac{{V_2}^2}{2g}$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{{V_2}^2}{2g} - \frac{{V_1}^2}{2g} \qquad \text{---------------(1)}$$

Therefore

$\frac{P_1}{\rho g} - \frac{P_2}{\rho g}$ is the difference of pressure heads at section 1 and 2 and it is equal to h. So, substituting this value of h in equation (1), we get

Flow of Fluids

Expression for the rate of flow through venturimeter:-

$$h = \frac{V_2^{\ 2}}{2g} - \frac{V_1^{\ 2}}{2g} \qquad \text{---------------(2)}$$

Now applying continuity equation at section 1 and 2

$$A_1V_1 = A_2V_2$$

$$V_1 = \frac{A_2V_2}{A_1}$$

Substituting this value of V_1 in equation (2) and solving, we get

$$V_2 = A_1 \frac{\sqrt{2gh}}{\sqrt{A_1^{\ 2} - A_2^{\ 2}}}$$

Discharge $Q = A_2V_2$

Substituting this value of V_2 in above equation and solving, we get

$$Q = A_1A_2 \frac{\sqrt{2gh}}{\sqrt{A_1^{\ 2} - A_2^{\ 2}}}$$

Flow of Fluids

Q is the theoretical discharge under ideal conditions. Actual discharge will be less than the theoretical discharge. The actual discharge is given by the formula

$$Q = C_d A_1 A_2 \frac{\sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$$

Where C_d is the coefficient of venturimeter, and its value is less than 1.

Expression of 'h' given by differential U-tube manometer:

Case 1:The liquid in the manometer is heavier than the liquid flowing through the pipe. **h_m = difference of the liquid columns in U tube**

$h = h_m[\frac{S_h}{S_0} - 1]$ where S_h : Specific gravity of the heavier liquid

S_0 : Specific gravity of the flowing liquid

Case 2: The liquid in the manometer is lighter than the liquid flowing through the pipe.

$h = h_m[1 - \frac{S_L}{S_0}]$ where S_L : Specific gravity of the lighter liquid

Flow of Fluids

Problem [redacted] *An oil of sp. gr. 0.8 is flowing through a venturimeter having inlet diameter 20 cm and throat diameter 10 cm. The oil-mercury differential manometer shows a reading of 25 cm. Calculate the discharge of oil through the horizontal venturimeter. Take $C_d = 0.98$.*

Solution. Given :

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_h = 13.6$

Reading of differential manometer, $x = 25$ cm

$\therefore$ Difference of pressure head, $h = x\left[\dfrac{S_h}{S_o} - 1\right]$

$$= 25\left[\frac{13.6}{0.8} - 1\right] \text{ cm of oil} = 25\,[17 - 1] = 400 \text{ cm of oil.}$$

Dia. at inlet, $d_1 = 20$ cm

$\therefore$ $a_1 = \dfrac{\pi}{4} d_1^2 = \dfrac{\pi}{4} \times 20^2 = 314.16 \text{ cm}^2$

$d_2 = 10$ cm

Flow of Fluids

$\therefore \quad a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times 20^2 = 314.16 \text{ cm}^2$

$d_2 = 10 \text{ cm}$

$\therefore \quad a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$

$C_d = 0.98$

$\therefore$ The discharge Q is given by equation (6.8)

or

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - 7a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 400}$$

$$= \frac{21421375.68}{\sqrt{98696 - 6168}} = \frac{21421375.68}{304} \text{ cm}^3/\text{s}$$

$= 70465 \text{ cm}^3/\text{s} =$ **70.465 litres/s. Ans.**

Flow of Fluids

Problem [redacted] *A 30 cm × 15 cm venturimeter is inserted in a vertical pipe carrying water, flowing in the upward direction. A differential mercury manometer connected to the inlet and throat gives a reading of 20 cm. Find the discharge. Take $C_d = 0.98$.*

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

$\therefore$ $a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

$\therefore$ $a_2 = \frac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$

$$h = x\left[\frac{S_l}{S_o} - 1\right] = 20\left[\frac{13.6}{1.0} - 1.0\right] = 20 \times 12.6 = 252.0 \text{ cm of water}$$

$C_d = 0.98$

Discharge, $$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 981 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.3 - 31222.9}} = \frac{86067593.36}{684.4}$$

$= 125756 \text{ cm}^3/\text{s} = $ **125.756 lit/s. Ans.**

Flow of Fluids

Orifice Meter:

Orifice meter: is a device used for measuring the rate of flow of a fluid flowing through a pipe.

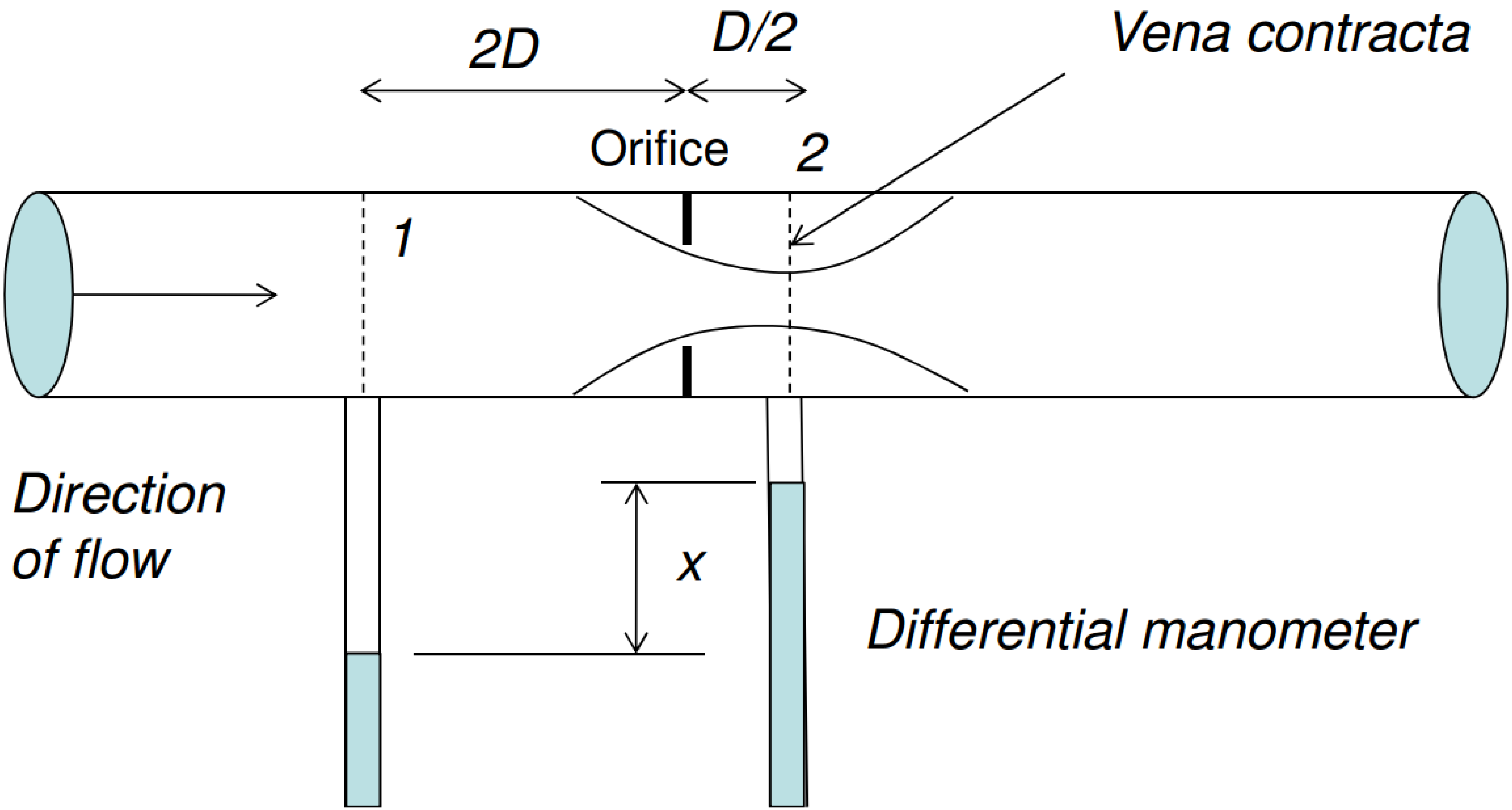
- It is a cheaper device as compared to venturimeter. This also work on the same principle as that of venturimeter.
- It consists of flat circular plate which has a circular hole, in concentric with the pipe. This is called orifice.
- The diameter of orifice is generally 0.5 times the diameter of the pipe (D), although it may vary from 0.4 to 0.8 times the pipe diameter.

Let

D_1 = diameter at section 1, P_1 = pressure at section 1, V_1 = velocity at section 1, A_1 = area at section 1.

Similarly D_2 = diameter at section 2, P_2 = pressure at section 2, V_2 = velocity at section 2, A_2 = area at section 2.

Flow of Fluids



Flow of Fluids

Applying Bernoulli's equation at sections 1 and 2

$$\frac{P_1}{\rho g} + \frac{V_1^{\ 2}}{2g} + \text{Z} = \frac{P_2}{\rho g} + \frac{V_2^{\ 2}}{2g} + \text{Z}$$

As the pipe is horizontal, so $Z_1 = Z_2$

$$\frac{P_1}{\rho g} + \frac{V_1^{\ 2}}{2g} = \frac{P_2}{\rho g} + \frac{V_2^{\ 2}}{2g}$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^{\ 2}}{2g} - \frac{V_1^{\ 2}}{2g} \qquad \text{---------------(1)}$$

Therefore

$\frac{P_1}{\rho g} - \frac{P_2}{\rho g}$ is the difference of pressure heads at section 1 and 2 and it is equal to h. So, substituting this value of h in equation (1), we get

$$\text{h} = \frac{V_2^{\ 2}}{2g} - \frac{V_1^{\ 2}}{2g} \qquad \text{---------------(2)}$$

$$V_2 = \sqrt{2gh + V_1^{\ 2}}$$

Where h is the differential head.

Flow of Fluids

Let A_0 is the area of the orifice.

Coefficient of contraction, $C_C = \frac{A_2}{A_0}$

Now applying continuity equation at section 1 and 2

$$A_1V_1 = A_2V_2$$

$$V_1 = \frac{C_C\,A_0}{A_1}V_2$$

Hence $V_2 = \dfrac{\sqrt{2gh}}{\sqrt{1-\frac{A_0^{\;2}}{A1^2}C_c^{\;2}}}$

Thus Discharge, $Q = A_2V_2 = C_c\,A_0\,V_2 = C_cA_0\,\dfrac{\sqrt{2gh}}{\sqrt{1-\frac{A_0^{\;2}}{A1^2}C_c^{\;2}}}$

$Q = A_0A_1\,\dfrac{\sqrt{2gh}}{\sqrt{A_1^{\;2}-A_0^{\;2}}}$ is called theoretical discharge.

Flow of Fluids

$Q_{Act} = C_d A_0 A_1 \frac{\sqrt{2gh}}{\sqrt{A12 - A02}}$ is called Actual discharge.

Where C_d is known as Coefficient of discharge. Whose value lies between 0.6 – 0.7

Flow of Fluids

Problem — *An orifice meter with orifice diameter 10 cm is inserted in a pipe of 20 cm diameter. The pressure gauges fitted upstream and downstream of the orifice meter gives readings of 19.62 N/cm² and 9.81 N/cm² respectively. Co-efficient of discharge for the meter is given as 0.6. Find the discharge of water through pipe.*

Solution. Given :

Dia. of orifice, $d_0 = 10$ cm

∴ Area, $a_0 = \dfrac{\pi}{4}(10)^2 = 78.54 \text{ cm}^2$

Dia. of pipe, $d_1 = 20$ cm

∴ Area, $a_1 = \dfrac{\pi}{4}(20)^2 = 314.16 \text{ cm}^2$

$p_1 = 19.62 \text{ N/cm}^2 = 19.62 \times 10^4 \text{ N/m}^2$

∴ $\dfrac{p_1}{\rho g} = \dfrac{19.62 \times 10^4}{1000 \times 9.81} = 20$ m of water

Flow of Fluids

Similarly $$\frac{p_2}{\rho g} = \frac{9.81 \times 10^4}{1000 \times 9.81} = 10 \text{ m of water}$$

$$h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 20.0 - 10.0 = 10 \text{ m of water} = 1000 \text{ cm of water}$$

$$C_d = 0.6$$

The discharge, Q is given by equation (6.13)

$$Q = C_d \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

$$= 0.6 \times \frac{78.54 \times 314.16}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 1000}$$

$$= \frac{20736838.09}{304} = 68213.28 \text{ cm}^3\text{/s} = \mathbf{68.21 \text{ litres/s. Ans.}}$$

Flow of Fluids

Problem *An orifice meter with orifice diameter 15 cm is inserted in a pipe of 30 cm diamet The pressure difference measured by a mercury oil differential manometer on the two sides of orifice meter gives a reading of 50 cm of mercury. Find the rate of flow of oil of sp. gr. 0.9 when co-efficient of discharge of the meter = 0.64.*

Solution. Given :

Dia. of orifice, $d_0 = 15$ cm

$\therefore$ Area, $a_0 = \frac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$

Dia. of pipe, $d_1 = 30$ cm

$\therefore$ Area, $a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Sp. gr. of oil, $S_0 = 0.9$

Reading of diff. manometer, $x = 50$ cm of mercury

Flow of Fluids

Problem *An orifice meter with orifice diameter 15 cm is inserted in a pipe of 30 cm diamet The pressure difference measured by a mercury oil differential manometer on the two sides of orifice meter gives a reading of 50 cm of mercury. Find the rate of flow of oil of sp. gr. 0.9 when co-efficient of discharge of the meter = 0.64.*

Solution. Given :

Dia. of orifice, $d_0 = 15$ cm

$\therefore$ Area, $a_0 = \frac{\pi}{4}(15)^2 = 176.7\ \text{cm}^2$

Dia. of pipe, $d_1 = 30$ cm

$\therefore$ Area, $a_1 = \frac{\pi}{4}(30)^2 = 706.85\ \text{cm}^2$

Sp. gr. of oil, $S_0 = 0.9$

Reading of diff. manometer, $x = 50$ cm of mercury

$\therefore$ Differential head, $h = x\left[\frac{S_g}{S_o} - 1\right] = 50\left[\frac{13.6}{0.9} - 1\right]$ cm of oil

Flow of Fluids

$$= 50 \times 14.11 = 705.5 \text{ cm of oil}$$

$$C_d = 0.64$$

$\therefore$ The rate of the flow, Q is given equation (6.13)

$$Q = C_d \cdot \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

$$= 0.64 \times \frac{176.7 \times 706.85}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 981 \times 705.5}$$

$$= \frac{94046317.78}{684.4} = 137414.25 \text{ cm}^3\text{/s} = \mathbf{137.414 \text{ litres/s. Ans.}}$$

Flow of Fluids

- Pitot Tube:

Pitot Tube is a device used for measuring the velocity of flow at any point in a pipe or a channel.

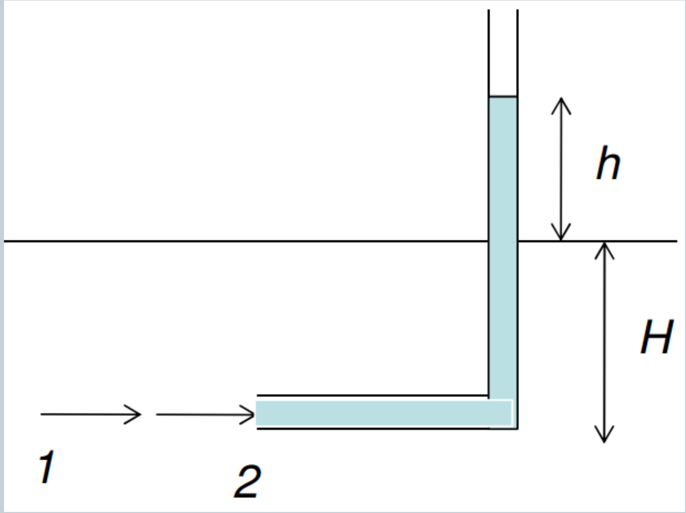
Principle: If the velocity at any point decreases, the pressure at that point increases due to the conservation of the kinetic energy into pressure energy.

In simplest form, the pitot tube consists of a glass tube, bent at right angles.

Let P_1= Pressure at section 1
P_2= Pressure at section 2
V_1= Velocity at section 1
V_2= Velocity at section 2
H= depth of tube in the liquid
h= rise of liquid in the tube above the free surface

Point 2 is just at the inlet of the pitot tube

Point 1 is far away from the tube



Flow of Fluids

Applying Bernoulli's equation at sections 1 and 2

$$\frac{P_1}{\rho g} + \frac{V_1^{\ 2}}{2g} + Z = \frac{P_2}{\rho g} + \frac{V_2^{\ 2}}{2g} + Z$$

As the pipe is horizontal, so $Z_1 = Z_2$

$$\frac{P_1}{\rho g} + \frac{V_1^{\ 2}}{2g} = \frac{P_2}{\rho g} + \frac{V_2^{\ 2}}{2g}$$

$\frac{P_1}{\rho g}$= Pressure head at section 1= H

$\frac{P_2}{\rho g}$= Pressure head at section 2= h + H

Substituting these values, we get

$H + \frac{V_1^{\ 2}}{2g} = h + H$

$V_1 = \sqrt{2gh}$ is called theoretical velocity.

Actual velocity $V_{1(Act)} = C_v\sqrt{2gh}$ where C_v is known as Coefficient of velocity

Flow of Fluids

Velocity of flow in a pipe by pitot-tube. For finding the velocity at any point in a pipe by pitot-tube, the following arrangements are adopted :

1. Pitot-tube along with a vertical piezometer tube as shown in Fig.
2. Pitot-tube connected with piezometer tube as shown in Fig.
3. Pitot-tube and vertical piezometer tube connected with a differential *U*-tube manometer as shown in Fig.

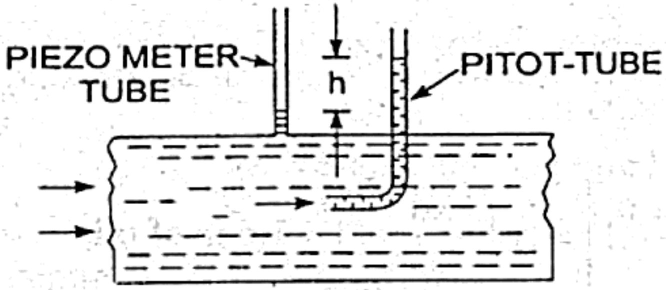


Fig.

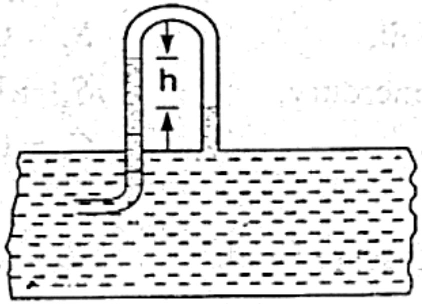
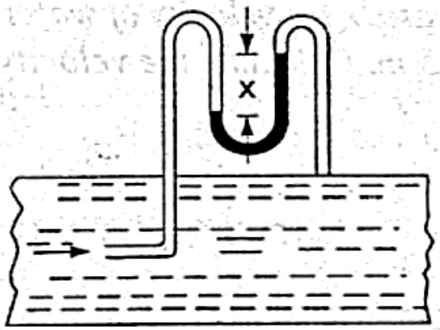
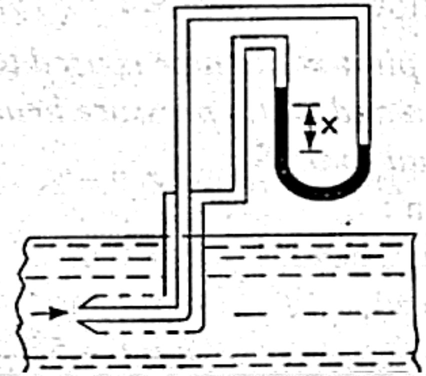


Fig.





Flow of Fluids

4. Pitot-static tube, which consists of two circular concentric tubes one inside the other with some annular space in between as shown in Fig. 6.17. The outlet of these two tubes are connected to the differential manometer where the difference of pressure head 'h' is measured by knowing the difference of the levels of the manometer liquid say x. Then $h = x\left[\frac{S_g}{S_o} - 1\right]$.

Problem *A pitot-static tube placed in the centre of a 300 mm pipe line has one orifice pointing upstream and other perpendicular to it. The mean velocity in the pipe is 0.80 of the central velocity. Find the discharge through the pipe if the pressure difference between the two orifices is 60 mm of water. Take the co-efficient of pitot tube as $C_v = 0.98$.*

Solution. Given :

Dia. of pipe, $d = 300\text{ mm} = 0.30\text{ m}$

Diff. of pressure head, $h = 60\text{ mm of water} = .06\text{ m of water}$

$C_v = 0.98$

Mean velocity, $\overline{V} = 0.80 \times \text{Central velocity}$

Central velocity is given by equation (6.14)

$$= C_v \sqrt{2gh} = 0.98 \times \sqrt{2 \times 9.81 \times .06} = 1.063\text{ m/s}$$

Flow of Fluids

$\therefore$ $$\bar{V} = 0.80 \times 1.063 = 0.8504 \text{ m/s}$$

Discharge, $$Q = \text{Area of pipe} \times \bar{V}$$

$$= \frac{\pi}{4} d^2 \times \bar{V} = \frac{\pi}{4} (.30)^2 \times 0.8504 = \mathbf{0.06\ m^3/s.\ Ans.}$$

Problem *Find the velocity of the flow of an oil through a pipe, when the difference of mercury level in a differential U-tube manometer connected to the two tappings of the pitot-tube is 100 mm. Take co-efficient of pitot-tube 0.98 and sp. gr. of oil = 0.8.*

Solution. Given :

Diff. of mercury level, $x = 100 \text{ mm} = 0.1 \text{ m}$

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_g = 13.6$

$C_v = 0.98$

Diff. of pressure head, $$h = x\left[\frac{S_g}{S_o} - 1\right] = .1\left[\frac{13.6}{0.8} - 1\right] = 1.6 \text{ m of oil}$$

$\therefore$ Velocity of flow $$= C_v\sqrt{2gh} = 0.98\sqrt{2 \times 9.81 \times 1.6} = \mathbf{5.49\ m/s.\ Ans.}$$

Flow of Fluids

Problem *A sub-marine moves horizontally in sea and has its axis 15 m below the surface of water. A pitot-tube properly placed just in front of the sub-marine and along its axis is connected to the two limbs of a U-tube containing mercury. The difference of mercury level is found to be 170 mm. Find the speed of the sub-marine knowing that the sp. gr. of mercury is 13.6 and that of sea-water is 1.026 with respect of fresh water.* (A.M.I.E., Winter, 1975)

Solution. Given :

Diff. of mercury level, $x = 170 \text{ mm} = 0.17 \text{ m}$

Sp. gr. of mercury, $S_g = 13.6$

Sp. gr. of sea-water, $S_o = 1.026$

$$\therefore \quad h = x\left[\frac{S_g}{S_o} - 1\right] = 0.17\left[\frac{13.6}{1.026} - 1\right] = 2.0834 \text{ m}$$

$$\therefore \quad V = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 2.0834} = 6.393 \text{ m/s}$$

$$= \frac{6.393 \times 60 \times 60}{1000} \text{ km/hr} = \mathbf{23.01 \text{ km/hr. Ans.}}$$

Chapter 5: Impact of Jet

Impact of Jet:

- A jet of water issuing from a nozzle has a velocity and hence it possesses a kinetic energy.
- If this jet strikes a plate, then it is said to have an impact on the plate. The jet will exert a force on the plate which it strikes. This force is called a dynamic force exerted by the jet.
- This force is due to the change in the momentum of the jet because of the impact. This force is equal to the rate of change of momentum i.e.,
- The force is equal to (mass striking the plate per second) x (change in velocity).

Impact of Jet

Impulse Momentum Principle:

- The impulse-momentum theorem states that the change in momentum of an object equals the impulse applied to it.
- The impulse-momentum theorem is logically equivalent to Newton's second law of motion (the force law).
- From Newton's 2nd Law: $F = m\,a = m\,(V_1 - V_2) / t$
- Impulse of a force is given by the change in momentum caused by the force on the body.

$$Ft = \text{Impulse} = mV_1 - mV_2 = \text{Initial Momentum} - \text{Final Momentum}$$

Force exerted by jet on the plate in the direction of jet, $F = m\,(V_1 - V_2) / t$

$$= (\text{Mass} / \text{Time})\,(\text{Initial Velocity} - \text{Final Velocity})$$

$$= (\rho Q)\,(V_1 - V_2) = (\rho a V)\,(V_1 - V_2)$$

Impact of Jet

Force Exerted By Fluid Jet On Flat Plate:

The following cases of the impact of jet, i.e. the force exerted by the jet on a plate will be considered:

1. Force exerted by the jet on a stationary plate
   a) Plate is vertical to the jet
   b) Plate is inclined to the jet
   c) Plate is curved
2. Force exerted by the jet on a moving plate
   a) Plate is vertical to the jet
   b) Plate is inclined to the jet
   c) Plate is curved

Impact of Jet

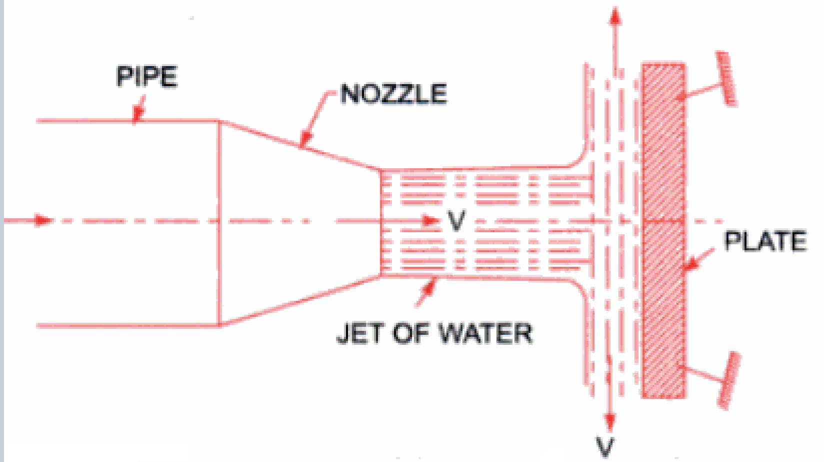
Force Exerted By Fluid Jet on a Stationary vertical Flat Plate:

Consider a jet of water coming out from the nozzle strikes the vertical plate.

V = velocity of jet

d = diameter of the jet

a = area of x-section of the jet



Force exerted by jet on vertical plate.

Impact of Jet

Force Exerted By Fluid Jet on a Stationary vertical Flat Plate:

- The force exerted by the jet on the plate in the direction of jet.

  F_x = Rate of change of momentum in the direction of force=(initial momentum – final momentum) /time

  = (mass x initial velocity – mass x final velocity) / time

  = mass/time (initial velocity – final velocity)

  = (density x discharge) x (velocity of jet before striking) x (velocity of jet before striking – final velocity of jet after striking)

  $= (\rho\ Q)\ (V_1 - V_2) = (\rho\ a\ V)\ (V\text{-}0) = (\rho a V^2)$ (because V_2= zero)

- Work done by the jet on a flat stationary plate = force × distance in the direction of force /time = $F_x \times$ zero=zero

  Kinetic energy of the jet= $\frac{1}{2}\ mV^2 = \frac{1}{2}\ \boldsymbol{\rho} AV^3$

  Efficiency of the jet = work done by the jet / kinetic energy of the jet= zero

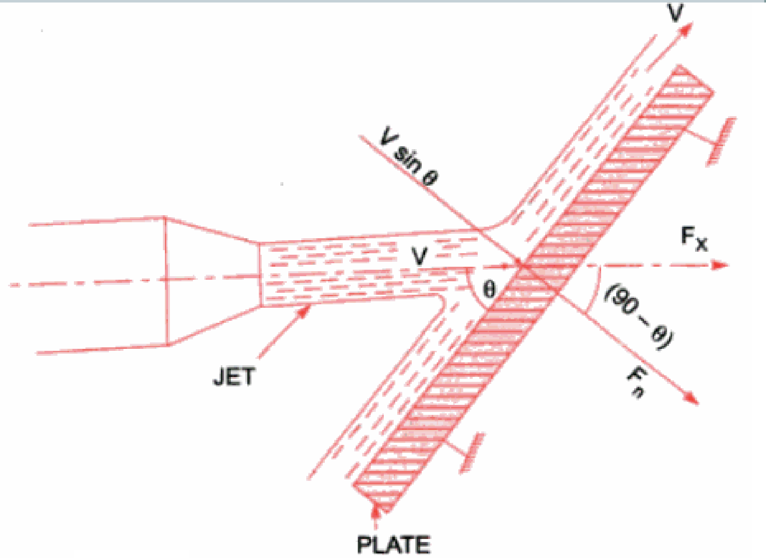
Impact of Jet

Force of Jet Impinging on An Inclined Fixed Plate:

Consider a jet of water impinging normally on a fixed plate

The force exerted by the jet on the plate in the direction of jet.

Let θ = Angle at which the plate is inclined
with the jet



Jet striking stationary inclined plate.

Impact of Jet

Force Exerted By Fluid Jet on an inclined fixed Plate:

- The force exerted by the jet in a direction normal to the plate.

F_n = Rate of change of momentum in the direction of force=(initial momentum – final momentum) /time

= mass/time (initial velocity – final velocity)

= (density x discharge) x (velocity of jet before striking) x (velocity of jet before striking – final velocity of jet after striking)

$= (\rho Q)(V_1 - V_2) = (\rho aV)(V\sin\theta - 0) = (\rho aV^2\sin\theta)$ (because V_2= zero)

The force exerted by the jet in a direction of the flow= $F_x = F_n\sin\theta = \rho aV^2\sin^2\theta$

The force exerted by the jet in a direction normal to flow= $F_y = F_n\cos\theta = \rho aV^2\sin\theta\cos\theta = (\rho aV^2\sin2\theta)/2$

Work done by the jet on a flat stationary plate = force × distance in the direction of force /time = F_x × zero=zero

Kinetic energy of the jet= $\frac{1}{2} mV^2 = \frac{1}{2} \boldsymbol{\rho} AV^3$

Efficiency of the jet = work done by the jet / kinetic energy of the jet= zero

Impact of Jet

- Problems:

Q1) Find the force exerted by a jet of water of diameter 75 mm on a stationary flat plate, when the jet strikes the plate normally with a velocity of 20 m/s.

Solution: Given:

i) Diameter of jet, d = 75mm = 0.075mm

ii) Area, $a = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.075)^2 = 0.004417m^2$

iiii) Velocity of jet, v = 20m/s

The force exerted by the jet of water on a stationary plate is given by,

$$F = \rho a v^2$$

$$F = 1000 \times 0.004417 \times 20^2 = 1766.8N$$

Impact of Jet

Q2) A jet of water of diameter 50 mm strikes a fixed blade in such a way that the angle between the plate and the jet is 30°. The force exerted in the direction of jet is 1471.5N. Determine the rate of flow of water.

Solution:

Given:

i) Diameter of jet, d = 50mm = 0.05m

ii) Area, $a = \frac{\pi}{4} \times (0.05)^2 = 0.00163m^2$

iii) Angle,$\theta = 30^\circ$

iv) Force in the direction of jet,

$F_x = 1471.5N$

As force is given in (N), ρ should be taken equal 1000 kg/m^3

$\therefore 1471.5 = 1000 \times 0.001963 \times v^2 \times \sin^2 30$

$\therefore v^2 = \frac{150}{0.05} = 3000$

$v = \sqrt{3000} = 54.77m/s$

Discharge, Q = Area $\times$ velocity

$Q = 0.001963 \times 54.77 = 0.1075m^3/s$

Impact of Jet

Force Exerted By Fluid Jet On a Moving vertical Flat Plate:

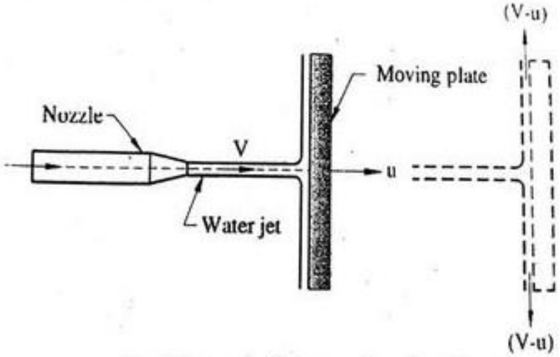
Consider a jet of water coming out from the nozzle strikes the vertical plate.

V = velocity of jet, d = diameter of the jet ,A = area of x-section of the jet

$\therefore$ Mass of water striking the plate per second

$= \rho A \times$ velocity with which jet strikes the plate

$= \rho A(V - u)$



Impact of jet on moving flat plate

Now, impact of water jet on moving plate in the direction of jet,

F_x = (Mass of jet strikes/sec) $\times$ [Initial relative velocity of jet $-$ Final relative velocity of jet in the direction of jet]

$$= \rho A(V - u) \times [(V - u) - (u - u)]$$

$$\therefore F_x = \rho A(V - u)^2$$

Impact of Jet

Work done by the jet on a flat moving plate = force × distance in the direction of force /time = $F_x \times u = \boldsymbol{\rho} A(V-u)^2 \times u$

Efficiency of the jet = work done by the jet per second / kinetic energy of the jet per second

Kinetic energy of the jet= $\frac{1}{2} mV^2 = \frac{1}{2} \boldsymbol{\rho} AV^3$

Hydraulic efficiency η= work done by the jet/ kinetic energy of the jet

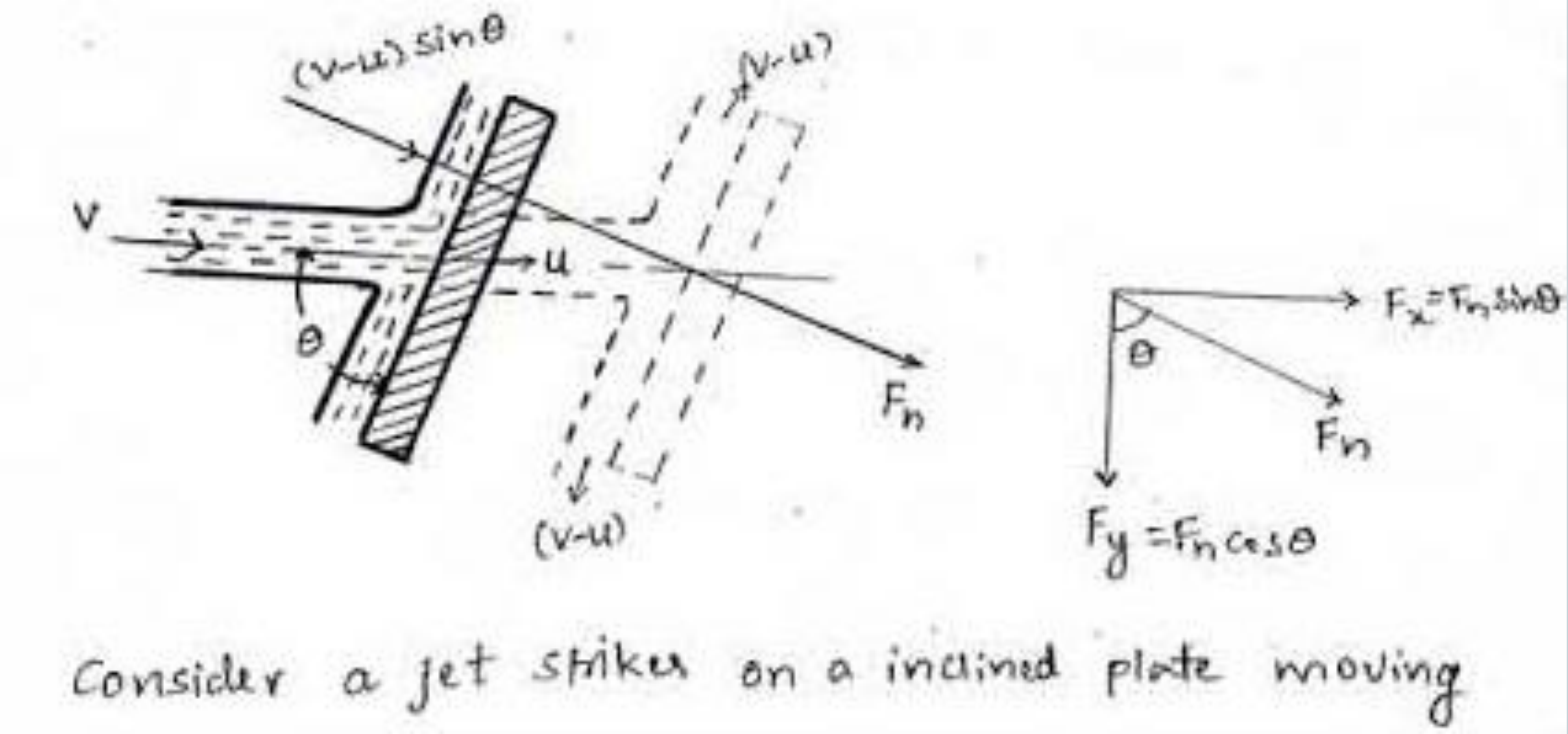
$$= \boldsymbol{\rho} A(V-u)^2 \times u / \tfrac{1}{2} \boldsymbol{\rho} AV^3$$

Impact of Jet

Force Exerted By Fluid Jet On a Moving inclined Flat Plate:

Consider a jet of water coming out from the nozzle strikes the vertical plate.

V = velocity of jet, d = diameter of the jet ,A = area of x-section of the jet



Impact of Jet

Force Exerted By Fluid Jet on a moving inclined Plate:

Let the velocity of the jet and the vane be V and u in the same direction. Let the angle between the jet and the plate be θ. In this case mass of liquid striking the plate per second = $\rho a (V - u)$

Relative velocity normal to the plate before impact = $(V - u) \sin \theta$

Relative velocity normal to the plate after impact = 0

$\therefore$ Force exerted by the jet normal to the plate

F_n = Rate of change of momentum in the direction of force=(initial momentum – final momentum) /time = (mass x initial velocity – mass x final velocity) / time

$F_n = \rho a (V - u) [(V - u) \sin \theta - 0]$

$\therefore F_n = \rho a (V - u)^2 \sin \theta$ Newton

The force exerted by the jet in a direction of the flow= $F_x = F_n \sin\theta = \rho a (V - u)^2 \sin^2\theta$

The force exerted by the jet in a direction normal to flow= $F_y = F_n \cos\theta = \rho a (V - u)^2 \sin\theta \cos\theta = (\rho a (V - u)^2 \sin 2\theta)/2$

Impact of Jet

Problems:1 A jet of water of diameter 10 cm strikes a flat plate normally with a velocity of 15 m/sec. The plate is moving with a velocity of 6m/sec in the direction of the jet and away from the jet. Find:(1) the force exerted by the jet on the plate

(2) Work done by the jet on the plate per second (3) Find the power and efficiency of the jet.

Solution: (1)diameter of jet d= 10cm= 0.1m

Area a=3.142 d^2= 0.007854m^2

Velocity of jet V= 15m/sec

Velocity of the plate u=6m/sec

$$F_x = \rho a (V-u)^2 = 1000 \times .007854 \times (15-6)^2 N = 636.17N$$

(2) Work done per second by the jet

$$= F_x \times u = 636.17 \times 6 = 3817.02 Nm/sec.$$

Impact of Jet

(3) Power of the jet in kW= work done per second/1000= 3817.02/1000=3.817kW

Efficiency of the jet (η)= work done by the jet per second / kinetic energy of the jet per second

Kinetic energy of the jet= ½ mV^2= ½ $\rho A V^3 = \frac{1}{2} \times 1000 \times .007854 \times 15^3 = 13253.6 \mathrm{Nm/sec}$

Hydraulic efficiency $\eta =$ work done by the jet/ kinetic energy of the jet

$= 3817.02/13253.6 = 0.288 = 28.8\%$ Ans.

Impact of Jet

Problems:2 A 7.5 cm diameter jet having a velocity of 30 m/sec strikes a flat plate, the normal of which is inclined at 45^0 to the axis of the jet. Find the normal pressure on the plate: (1) when the plate is stationary (2) when the plate is moving with a velocity of 15 m/sec, and away from the jet. Also determine the power and efficiency of the jet when the plate is moving.

Solution : diameter d= 7.5 cm= 0.075m

Area =a= $3.142 \times (0.075)^2 = 0.004417 m^2$

Angle between the jet and plate $\theta = 90^0 - 45^0 = 45^0$

Velocity of jet, V=30m/sec

(1) When the plate is stationary, the normal force on then plate is

$$F_n = \rho a v^2 \sin\theta = 1000 \times 0.004417 \times 30^2 \times \sin 45 = 2810.96 N$$

(2) When the plate is moving with a velocity of 15 m/sec and away from the jet, the normal force on the plate is given by

$$F_n = \rho a (V-u)^2 \sin\theta = 1000 \times 0.004417 \times (30-15)^2 \times \sin 45 = 702.74 N$$

Impact of Jet

(3) The power and efficiency of the jet is obtained as :

Work done by the jet per second $= F_x \times u = F_n \sin\theta \times u = 702.74 \times \sin 45 \times 15 = 7453.5 \text{Nm/sec}$

Power in kW $= 7453.5/1000 = 7.453\text{kW}$

Efficiency of the jet (η)= work done by the jet per second / kinetic energy of the jet per second

Kinetic energy of the jet $= \frac{1}{2} mV^2 = \frac{1}{2} \rho A V^3 = \frac{1}{2} \times 1000 \times 0.004417 \times 30^3$

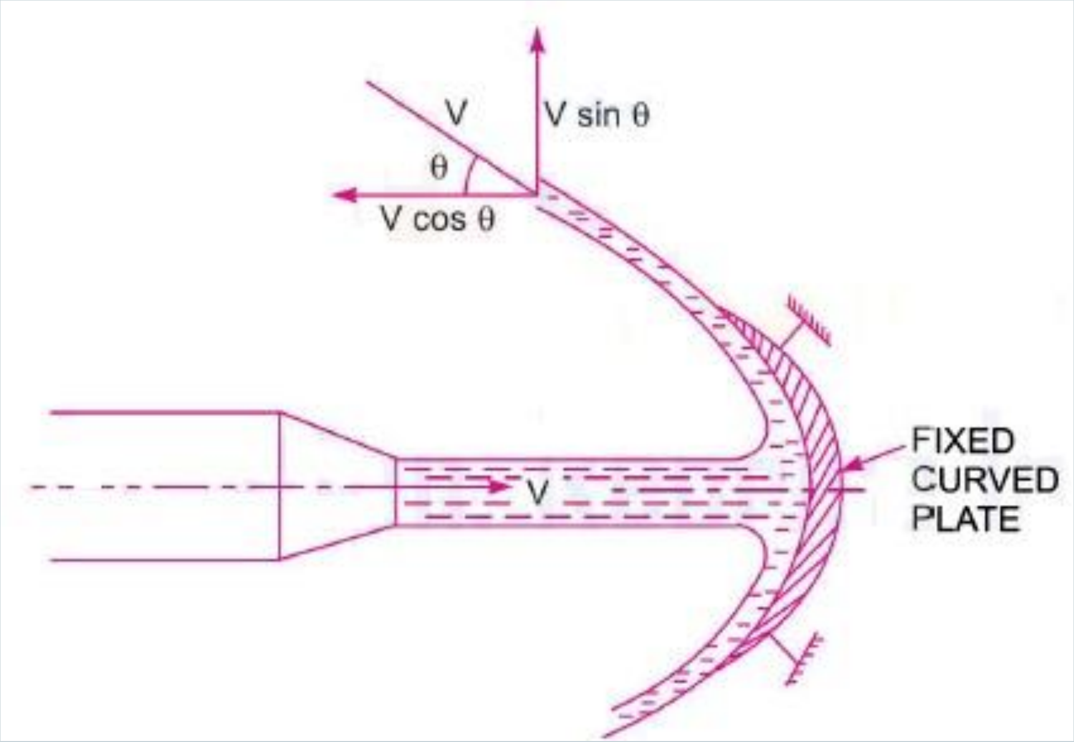
Hydraulic efficiency $\eta =$ work done by the jet/ kinetic energy of the jet

$= 0.1249 = 12.5\%$

Impact of Jet

Jet Striking a Symmetrical Curved Vane at Rest, at the Centre of the Vane:

In this case the jet after striking the vane at the center gets divided into two identical jets leaving the vane at the outer tips, with the velocity having the same magnitude as that of the striking jet if no resistances are offered by the vane.



Impact of Jet

The force exerted by the jet on the plate in the direction of jet.

F_x = Rate of change of momentum in the direction of force=(initial momentum – final momentum) /time

= (mass x initial velocity – mass x final velocity) / time

= mass/time (initial velocity – final velocity)

= (density x discharge) x (velocity of jet before striking) x (velocity of jet before striking – final velocity of jet after striking)

$= (\rho Q)(V_1 - V_2) = (\rho a V)(V-(-V\cos\theta)) = (\rho a V^2)(1+\cos\theta)$

- For the particular case, when the vane is flat, $\theta=90^0$ and $F_x = (\rho a V^2)$

  It is clearly seen that force exerted by the curved vane is greater than the force exerted by the flat plate.

- For another case, when the vane is semicircular , $\theta=0^0$ and $F_x = (2\rho a V^2)$

  In this case, force exerted by the curved vane is two times that of the force exerted by the flat plate.

Impact of Jet

Jet Striking a Symmetrical Curved Vane at Rest, at the tip of the Vane:

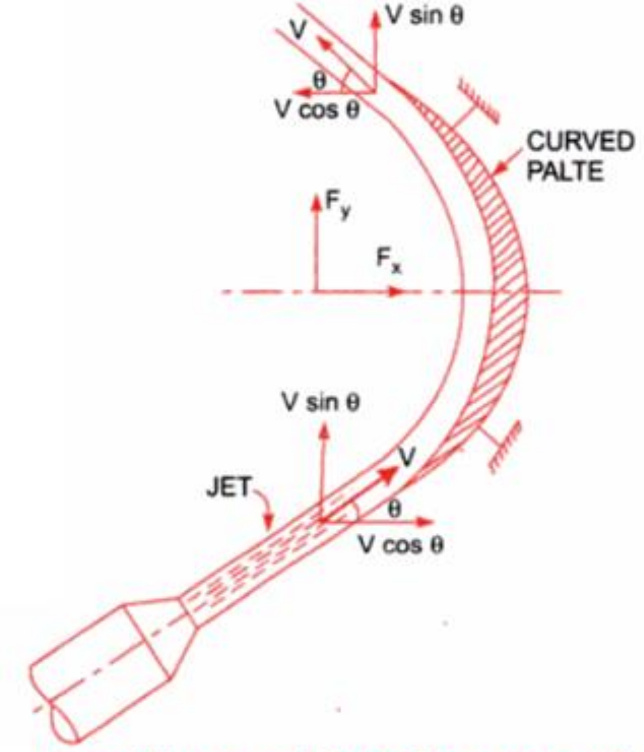
In this case the jet after striking the vane at the center gets divided into two identical jets leaving the vane at the outer tips, with the velocity having the same magnitude as that of the striking jet if no resistances are offered by the vane.

The force exerted by the jet on the plate in the direction of jet.

F_x = Rate of change of momentum in the direction of force=(initial momentum – final momentum) /time

$= (\rho Q)(V_1 - V_2) = (\rho a V)(V \cos\theta - (- V \cos\theta))$

$= (2\rho a V^2)(\cos\theta)$

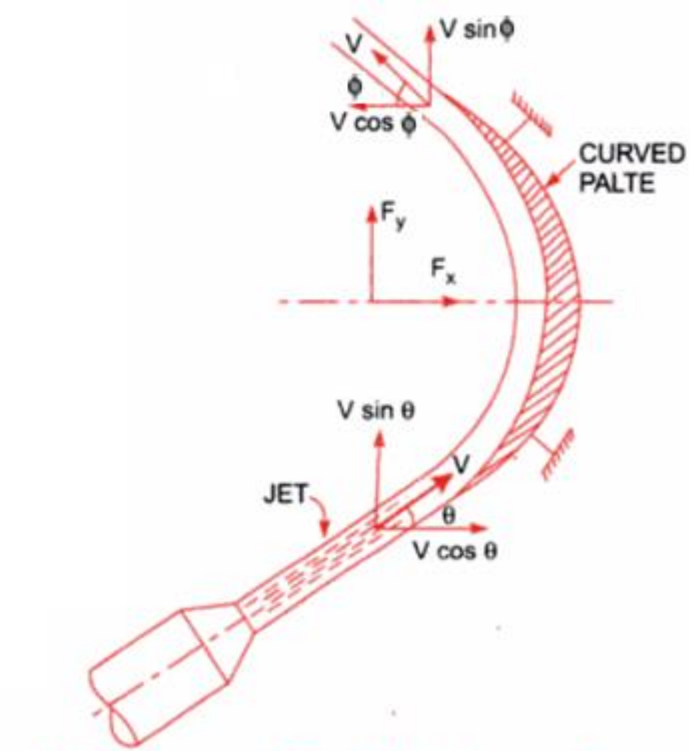


Jet striking curved fixed plate at one end.

Impact of Jet

Jet Striking an Unsymmetrical Curved Vane at Rest, at the tip of the Vane:

In this case the jet after striking the vane at the center gets divided into two identical jets leaving the vane at the outer tips, with the velocity having the same magnitude as that of the striking jet if no resistances are offered by the vane.



Jet striking curved fixed plate at one end.

The force exerted by the jet on the plate

in the direction of jet.

F_x = Rate of change of momentum in the direction

of force=(initial momentum – final momentum) /time

$= (\rho\ Q)\ (V_1 - V_2) = (\rho\ a\ V)\ (V \cos\theta - (- V \cos\phi))$

$= (\rho a V^2)\ (\cos\theta + \cos\phi)$

$F_y = (\rho a V^2)\ (\sin\theta - \sin\phi)$

Impact of Jet

A jet of water of diameter 50 mm moving with a velocity of 40 m/s, strikes a curved fixed symmetrical plate at the centre. Find the force exerted by the jet of water in the direction of the jet, if the jet is deflected through an angle of 120° at the outlet of the curved plate.

Solution:

Given:

i) Diameter of jet, d = 50mm = 0.05m

ii) Area, $a = \frac{\pi}{4}(0.05)^2 = 0.001963 m^2$

iii) Velocity, v = 40m/s

$\therefore$ To find θ, use equation

$180 - \theta$ = given angle

$$180 - \theta = 120$$

$$\therefore, \theta = 180 - 120 = 60^\circ$$

$$\therefore F_x = \rho a v^2 [1 + \cos\theta]$$

$$F_x = 1000 \times 0.001963 \times 40^2 \times [1 + \cos 60^\circ]$$

$$F_x = 4711.5 N$$

Impact of Jet

A jet of water of diameter 75 mm moving with a velocity of 30 m/s, strikes a curved fixed plate tangentially at one end at an angle of 30° to the horizontal, the jet leaves the plate at an angle of 20° to the horizontal, find the force exerted by a jet on the plate in the horizontal and vertical direction.

Solution: Given:

i) Diameter of jet, d = 75mm = 0.075m

ii) Area, $a = \frac{\pi}{4} \times (d^2)$

$$a = \frac{\pi}{4} \times (0.075)^2 = 0.004417m^2$$

iii) Velocity, v = 30m/s

iv) Angle made by the jet at inlet tip with horizontal, $\theta = 30^\circ$

v) Angle made by the jet at outlet tip with horizontal, $\phi = 20^\circ$

$$\therefore F_x = \rho a v^2 [\cos\theta + \cos\phi]$$

$$F_x = 1000 \times 0.004417 \times 30^2 \times [\cos 30^\circ + \cos 20^\circ]$$

$$F_x = 7178.2N$$

$$F_y = \rho a v^2 [\sin\theta - \sin\phi]$$

$$F_y = 1000 \times 0.004417 [\sin 30 - \sin 20] \times 30^2$$

$$F_y = 628.13N$$

Impact of Jet

Jet Striking a Symmetrical Curved moving vane, at the centre of the Vane:

In this case the jet after striking the vane at the center gets divided into two identical jets leaving the vane at the outer tips, with the velocity having the same magnitude as that of the striking jet if no resistances are offered by the vane.

The force exerted by the jet on the plate

in the direction of jet.

F_x = Rate of change of momentum in the direction

of force=(initial momentum – final momentum) /time

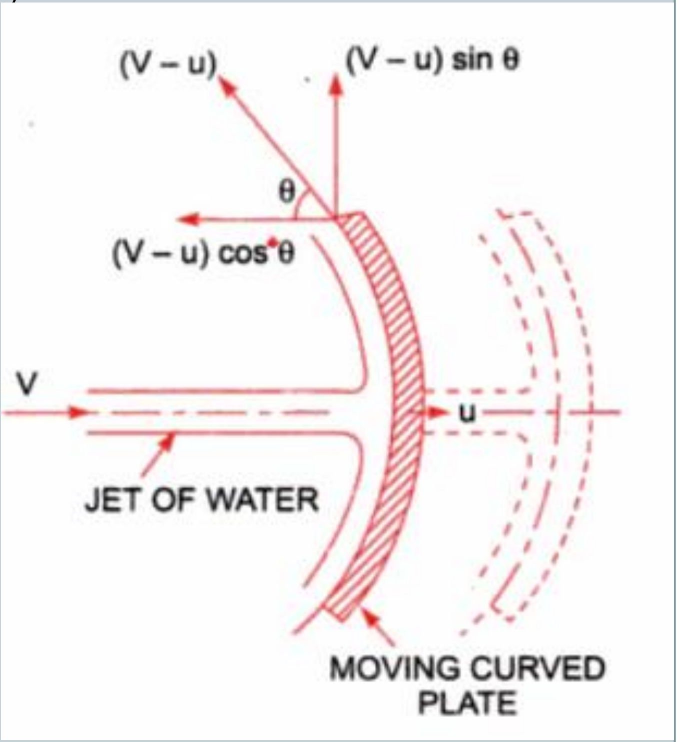
$= (\rho Q)(V_1 - V_2) = \rho a (V - u)((V\text{-}u)\text{-}(V\text{-}u)\cos\theta)$

$= \rho a (V - u)^2 (1+\cos\theta)$

Work done by the jet on a flat moving plate

= force × distance in the direction of force /time

$= F_x \times u = \rho a (V - u)^2 (1+\cos\theta) \times u$



Impact of Jet

Problem 17.14 *A jet of water of diameter 7.5 cm strikes a curved plate at its centre with a velocity of 20 m/s. The curved plate is moving with a velocity of 8 m/s in the direction of the jet. The jet is deflected through an angle of 165°. Assuming the plate smooth find :*

(i) Force exerted on the plate in the direction of jet, (ii) Power of the jet, and (iii) Efficiency of the jet.

Solution. Given :

Diameter of the jet, $d = 7.5 \text{ cm} = 0.075 \text{ m}$

∴ Area, $a = \frac{\pi}{4}(.075)^2 = 0.004417$

Velocity of the jet, $V = 20 \text{ m/s}$

Velocity of the plate, $u = 8 \text{ m/s}$

Angle of deflection of the jet, $= 165°$

∴ Angle made by the relative velocity at the outlet of the plate,

$$\theta = 180° - 165° = 15°.$$

(*i*) Force exerted by the jet on the plate in the direction of the jet is given by equation (17.17) as

$$= F_x = \rho a(V - u)^2 (1 + \cos\theta)$$

$$= 1000 \times .004417 \times (20 - 8)^2 [1 + \cos 15°] = \mathbf{1250.38 \text{ N. Ans.}}$$

(*ii*) Work done by the jet on the plate per second

$$= F_x \times u = 1250.38 \times 8 = 10003.04 \text{ N m/s}$$

∴ Power of the jet $= \frac{10003.04}{1000} = \mathbf{10 \text{ kW. Ans.}}$

(*iii*) Efficiency of the jet $= \frac{\text{Output}}{\text{Input}} = \frac{\text{Work done by jet/sec}}{\text{Kinetic energy of jet/sec}}$

$$= \frac{1250.38 \times 8}{\frac{1}{2}(\rho a V) \times V^2} = \frac{1250.38 \times 8}{\frac{1}{2} \times 1000 \times .004417 \times V^3}$$

$$= \frac{1250.38 \times 8}{\frac{1}{2} \times 1000 \times .004417 \times 20^3} = 0.564 = \mathbf{56.4\%.\ Ans}$$

Impact of Jet

Problem 17.15 *A jet of water from a nozzle is deflected through 60° from its original direction by a curved plate which it enters tangentially without shock with a velocity of 30 m/s and leaves with a mean velocity of 25 m/s. If the discharge from the nozzle is 0.8 kg/s, calculate the magnitude and direction of the resultant force on the vane, if the vane is stationary.*

Solution. Given :

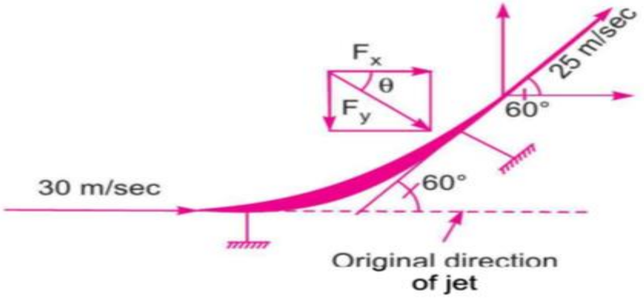


Fig. 17.13 (*a*)

Velocity at inlet, $V_1 = 30$ m/s

Velocity at outlet, $V_2 = 25$ m/s

Mass per second $= 0.8$ kg/s

Force in the direction of jet,

$$F_x = \text{Mass per second} \times (V_{1x} - V_{2x})$$

where V_{1x} = Initial velocity in the direction of x

$= 30$ m/s

V_{2x} = Final velocity in the direction of x

$$= 25 \cos 60^\circ = 25 \times \frac{1}{2} = 12.5 \text{ m/s}$$

$$\therefore \quad F_x = 0.8[30 - 12.5] = 0.8 \times 17.5 = 14.0 \text{ N}$$

Similarly, force normal to the jet,

$$F_y = \text{Mass per second} \times (V_{1y} - V_{2y})$$

$$= 0.8\ [0 - 25 \sin 60^\circ] = -17.32 \text{ N}$$

–ve sign means the force, F_y, is acting in the vertically downward direction.

$$\therefore \quad \text{Resultant force on the vane} = \sqrt{F_x^2 + F_y^2} = \sqrt{14^2 + (-17.32)^2} = \mathbf{22.27\ N.}\ \ \textbf{Ans.}$$

The angle made by the resultant with x-axis

$$\tan\theta = \frac{F_y}{F_x} = \frac{-17.32}{14.0} = -1.237$$

–ve sign means the angle θ is in the clockwise direction with x- axis as shown in Fig. 17.13 (*a*)

$$\therefore \quad \theta = \tan^{-1} 1.237 = \mathbf{51^\circ\ 2.86'}.\ \textbf{Ans.}$$

Impact of Jet

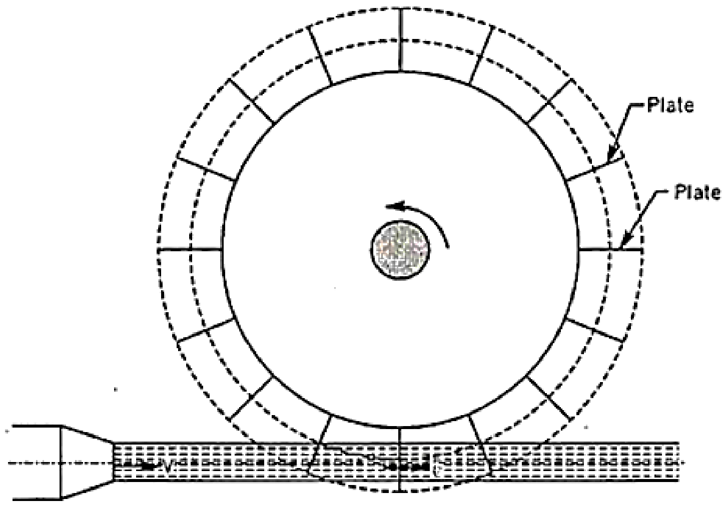
Direct Impact of a Jet on a Series of Flat Vanes Mounted on the Periphery of a Large Wheel:

In this case, a number of flat plates are radially mounted over a wheel. The wheel is supported over a shaft, with a suitable bearing to afford easy rotation of the wheel. The jet moving at a velocity strikes the plate in succession causing the wheel to rotate.

Velocity of the jet before striking the wheel = V

Velocity of the jet after striking the wheel

= v = Velocity of the plates at the impact point.



Impact of Jet

In this case, *force exerted by the jet on the wheel*

$$P = \text{Mass striking the wheel per second} \times \text{Change in velocity}$$

$$\therefore \quad P = \rho a V (V - v) \text{ Newton}$$

$\therefore$ *Work done per second*

$$= Pv \text{ Nm/sec} = \rho a V (V - v)\, v \text{ Nm/sec}$$

Energy supplied by the jet per second

$$= \frac{1}{2} M V^2 = \frac{1}{2} \rho a V V^2 = \frac{1}{2} \rho a V^3$$

$$\textit{Hydraulic efficiency} = \eta$$

$$= \frac{\text{Work done per second}}{\text{Kinetic energy of jet per second}} = \frac{\rho a V (V - v) v}{\frac{1}{2} \rho a V^3}$$

Impact of Jet

$$\eta = \frac{2(V-v)v}{V^2}$$

Condition for maximum hydraulic efficiency

We know, hydraulic efficiency, $\eta = \dfrac{2(V-v)v}{V^2}$

For η to be maximum, $\dfrac{d\eta}{dv} = 0$

$$\therefore \quad \left[\frac{2}{V^2}\right](V-2v) = 0$$

$$\therefore \quad V-2v = 0$$

$$\therefore \quad v = \frac{V}{2}$$

$$\therefore \quad \eta_{max} = \frac{2\left(V-\frac{V}{2}\right)\frac{V}{2}}{V^2} = 0.5 \text{ or } 50\%$$

Impact of Jet

The wheel with plates (vanes) provided as in this case and subjected to impact by a jet is called a water wheel. When the jet strikes the wheel at the bottom as in this case, the device is called an undershot water wheel. When the jet strikes the wheel at the top, the device is called an overshot water wheel.

Impact of Jet

18.6.1 Velocity Triangles and Work done for Pelton Wheel. Fig. 18.5 shows the shape of the vanes or buckets of the Pelton wheel. The jet of water from the nozzle strikes the bucket at the splitter, which splits up the jet into two parts. These parts of the jet, glides over the inner surfaces and comes out at the outer edge. Fig. 18.5 (*b*) shows the section of the bucket at *Z-Z*. The splitter is the inlet tip and outer edge of the bucket is the outlet tip of the bucket. The inlet velocity triangle is drawn at the splitter and outlet velocity triangle is drawn at the outer edge of the bucket, by the same method as explained in Chapter 17.

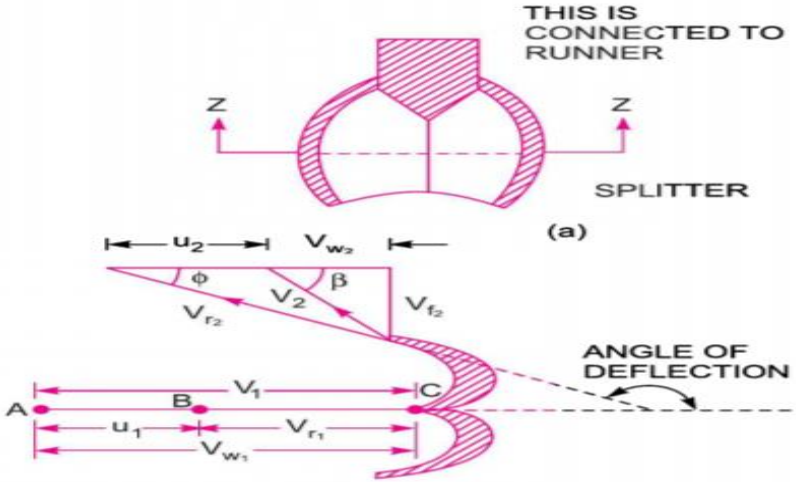


Fig. 18.5 *Shape of bucket.*

Let H = Net head acting on the Pelton wheel

$= H_g - h_f$

where H_g = Gross head and $h_f = \dfrac{4fLV^2}{D^* \times 2g}$

where D^* = Dia. of Penstock, N = Speed of the wheel in r.p.m.,

D = Diameter of the wheel, d = Diameter of the jet.

Then V_1 = Velocity of jet at inlet $= \sqrt{2gH}$ ...(18.7)

$$u = u_1 = u_2 = \frac{\pi DN}{60}.$$

The velocity triangle at inlet will be a straight line where

$$V_{r_1} = V_1 - u_1 = V_1 - u$$

$$V_{w_1} = V_1$$

$$\alpha = 0° \text{ and } \theta = 0°$$

From the velocity triangle at outlet, we have

$$V_{r_2} = V_{r_1} \text{ and } V_{w_2} = V_{r_2} \cos\phi - u_2.$$

The force exerted by the jet of water in the direction of motion is given by equation (17.19) as

$$F_x = \rho a V_1 [V_{w_1} + V_{w_2}] \qquad \text{...(18.8)}$$

As the angle β is an acute angle, +ve sign should be taken. Also this is the case of series of vanes, the mass of water striking is $\rho a V_1$ and not $\rho a V_{r_1}$. In equation (18.8), 'a' is the area of the jet which is given as

$$a = \text{Area of jet} = \frac{\pi}{4} d^2.$$

Now work done by the jet on the runner per second

$$= F_x \times u = \rho a V_1 [V_{w_1} + V_{w_2}] \times u \text{ Nm/s} \qquad \text{...(18.9)}$$

Power given to the runner by the jet

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{1000} \text{ kW} \qquad \text{...(18.10)}$$

Work done/s per unit weight of water striking/s

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\text{Weight of water striking/s}}$$

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\rho a V_1 \times g} = \frac{1}{g}\left[V_{w_1} + V_{w_2}\right] \times u \qquad \text{...(18.11)}$$

The energy supplied to the jet at inlet is in the form of kinetic energy and is equal to $\frac{1}{2} m V^2$

$\therefore$ K.E. of jet per second $= \frac{1}{2} (\rho a V_1) \times V_1^2$

$\therefore$ **Hydraulic efficiency**, $\eta_h = \dfrac{\text{Work done per second}}{\text{K.E. of jet per second}}$

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\frac{1}{2} (\rho a V_1) \times V_1^2} = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} \qquad \text{...(18.12)}$$

Now $\quad V_{w_1} = V_1,\ V_{r_1} = V_1 - u_1 = (V_1 - u)$

$\therefore \quad V_{r_2} = (V_1 - u)$

and $\quad V_{w_2} = V_{r_2} \cos\phi - u_2 = V_{r_2} \cos\phi - u = (V_1 - u) \cos\phi - u$

Substituting the values of V_{w_1} and V_{w_2} in equation (18.12),

$$\eta_h = \frac{2\left[V_1 + (V_1 - u)\cos\phi - u\right] \times u}{V_1^2}$$

$$= \frac{2\left[V_1 - u + (V_1 - u)\cos\phi\right] \times u}{V_1^2} = \frac{2(V_1 - u)\,[1 + \cos\phi]\,u}{V_1^2}. \qquad \text{...(18.13)}$$

Impact of Jet

The efficiency will be maximum for a given value of V_1 when

$$\frac{d}{du}(\eta_h) = 0 \qquad \text{or} \qquad \frac{d}{du}\left[\frac{2u(V_1 - u)(1+\cos\phi)}{V_1^2}\right] = 0$$

$$\text{or} \qquad \frac{(1+\cos\phi)}{V_1^2}\frac{d}{du}(2uV_1 - 2u^2) = 0 \qquad \text{or} \qquad \frac{d}{du}[2uV_1 - 2u^2] = 0 \qquad \left(\because \frac{1+\cos\phi}{V_1^2} \neq 0\right)$$

$$\text{or} \qquad 2V_1 - 4u = 0 \qquad \text{or} \qquad u = \frac{V_1}{2} \qquad \text{...(18.14)}$$

Equation (18.14) states that hydraulic efficiency of a Pelton wheel will be maximum when the velocity of the wheel is half the velocity of the jet of water at inlet. The expression for maximum efficiency will be obtained by substituting the value of $u = \dfrac{V_1}{2}$ in equation (18.13).

$$\therefore \qquad \text{Max. } \eta_h = \frac{2\left(V_1 - \frac{V_1}{2}\right)(1+\cos\phi) \times \frac{V_1}{2}}{V_1^2}$$

$$= \frac{2 \times \frac{V_1}{2}(1+\cos\phi)\frac{V_1}{2}}{V_1^2} = \frac{(1+\cos\phi)}{2}. \qquad \text{...(18.15)}$$

Chapter 6: Hydraulic Turbines

Classification of Hydraulic turbines:

1) Based on type of energy at inlet to the turbine:

- **Impulse Turbine** : The energy is in the form of kinetic form. e.g: Pelton wheel, Turbo wheel.
- **Reaction Turbine** : The energy is in both Kinetic and Pressure form. e.g: Tubular, Bulb, Propeller, Francis turbine.

2) Based on direction of flow of water through the runner:

- **Tangential flow:** water flows in a direction tangential to path of rotational, i.e. Perpendicular to both axial and radial directions.
- **Radial outward flow** e.g : Forneyron turbine.
- **Axial flow :** Water flows parallel to the axis of the turbine. e.g: Girard, Jonval, Kalpan turbine.
- **Mixed flow :** Water enters radially at outer periphery and leaves axially. e.g : Modern Francis turbine.

Hydraulic Turbines

3) Based on the head under which turbine works:

- High head, impulse turbine. e.g : Pelton turbine.
- Medium head, reaction turbine. e.g :Francis turbine.
- Low head, reaction turbine. e.g : Kaplan turbine, propeller turbine.

4) Based on the specific speed of the turbine:

- Low specific speed, impulse turbine. e.g : Pelton wheel.
- Medium specific speed, reaction turbine. e.g : Francis wheel.
- High specific speed, reaction turbine. e.g : Kaplan and Propeller turbine.

5) Based on the name of the originator:

- Impulse turbine – Pelton wheel, Girard, Banki turbine.
- Reaction turbine – Forneyron, Jonval, Francis, Dubs, Deriaze, Thomson Kalpan, Barker, Moody, Nagler, Bell.

Hydraulic Turbines

Difference between Impulse and Reaction turbine:

Impulse turbine	Reaction Turbine
The entire available energy of the water is first converted into kinetic energy.	The available energy of the water is not converted from one form to another.
The water flows through the nozzles and impings on the buckets, which are fixed to the outer periphery of the wheel.	The water is guided by the glide blades to flow over the moving vane.
The water impings on the buckets with KE	The water glides over the moving vanes with PE.
The pressure of the flowing water remains unchanged and is equal to the atmospheric pressure.	The pressure of the flowing water is reduced after gliding over the vane.
It is not essential that the wheel should run full.	It is essential that the wheel should always run full and kept full of water.
It is possible to regulate the flow without loss.	It is not possible to regulate the flow without loss.
Impulse Turbine has more hydraulic efficiency.	Reaction Turbine has relatively less efficiency.
Impulse Turbine operates at high water heads.	Reaction turbine operates at low and medium heads.
Example of Impulse turbine is Pelton wheel.	Examples of Reaction Turbine are Francis turbine, Kaplan and Propeller Turbine, Deriaz Turbine, Tubuler Turbine, etc.

Hydraulic Turbines

Hydraulic (Water) Turbines

- Basic working principle:

Hydraulic turbines convert the potential energy of water into mechanical work.

- Three most popular hydraulic turbines are :

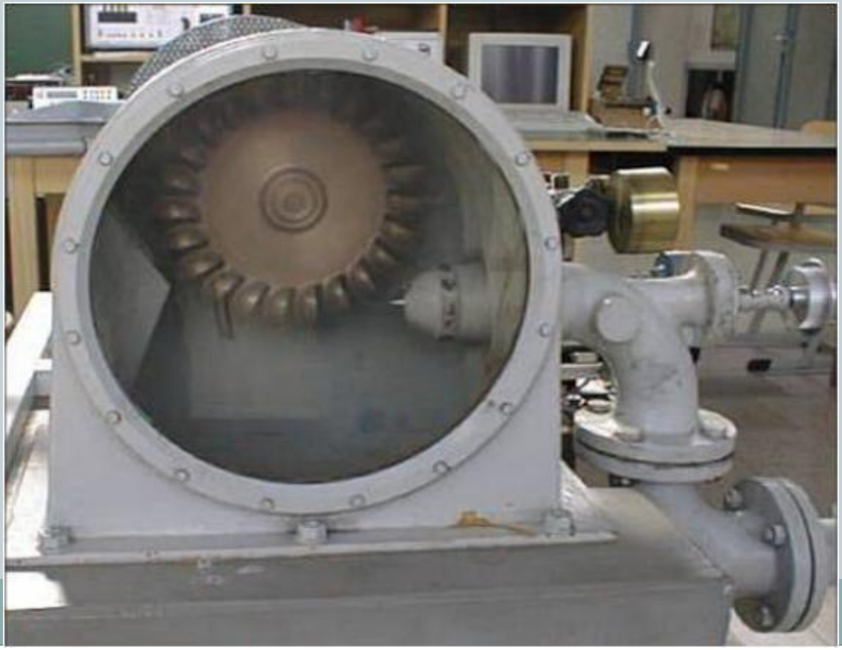
- Pelton wheel (Pelton turbine)
- Kaplan turbine (Propeller turbine)
- Francis turbine

Hydraulic Turbines

Pelton Wheel (Pelton turbine):

- This turbine is named after Lester A. Pelton (1829 – 1908) an American Engineer who developed it in the year 1880.
- Pelton wheel is a high head turbine. It is used with heads of more than 500 metres.

Note : A head is the distance the water falls before it strikes the turbine blades.



Hydraulic Turbines

- The flow of water is tangential to the runner. So it is a tangential flow impulse turbine.
- A Pelton's runner consists of a single wheel mounted on a horizontal shaft.
- Water falls towards the turbine through a pipe called penstock and flows through a nozzle.
- The high speed jet of water hits the buckets (vanes) on the wheel and causes the wheel to rotate.
- A spear rod which has a spear shaped end can be moved by a hand wheel.
- This movement controls the flow of water leaving the nozzle, before it strikes the bucket(vane)

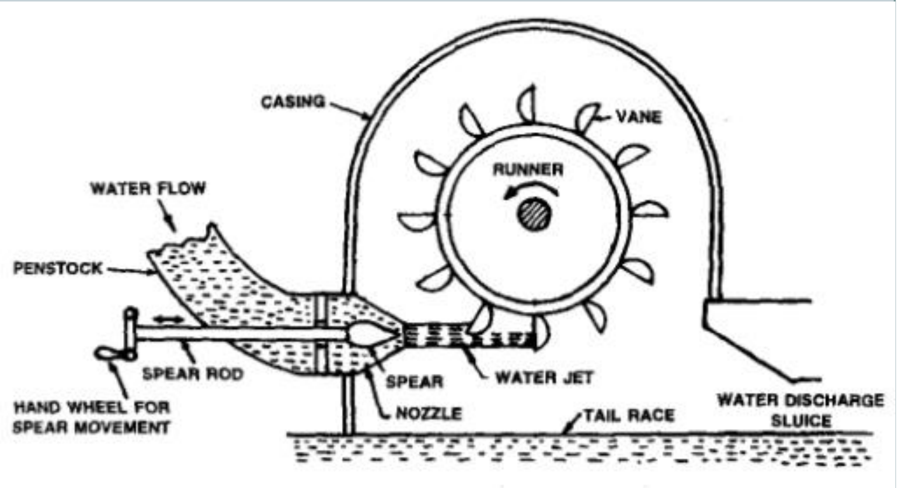


Fig. 2.32: VIEW SHOWING HOW THE WATER STRIKES THE BUCKET

Hydraulic Turbines

- The bucket or vane is so shaped that when the water strikes, it gets split into two and gives it an impulse force in the center of the bucket. This bucket is also known as splitter.

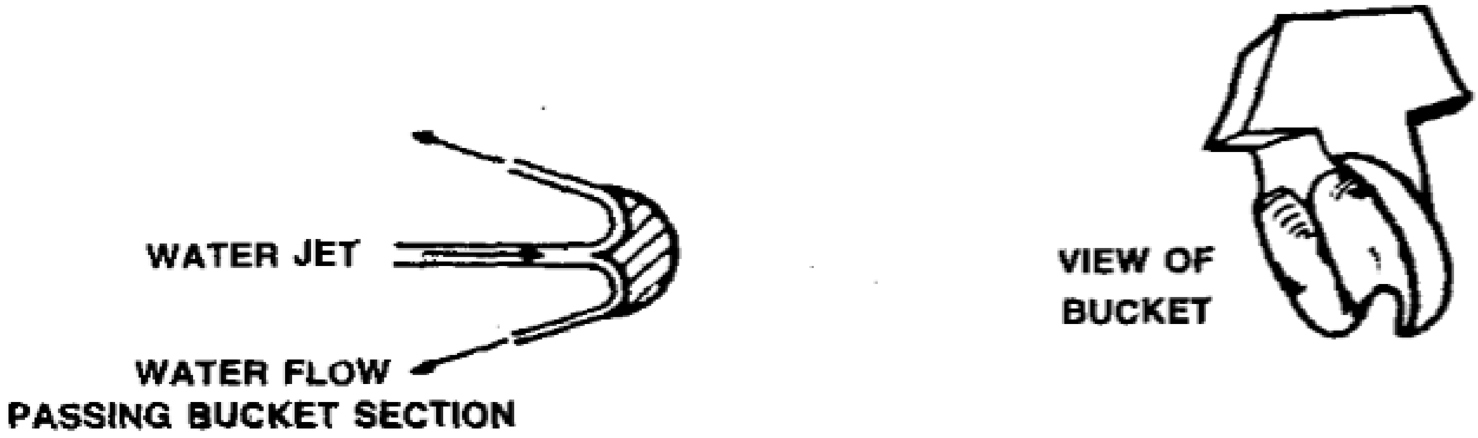
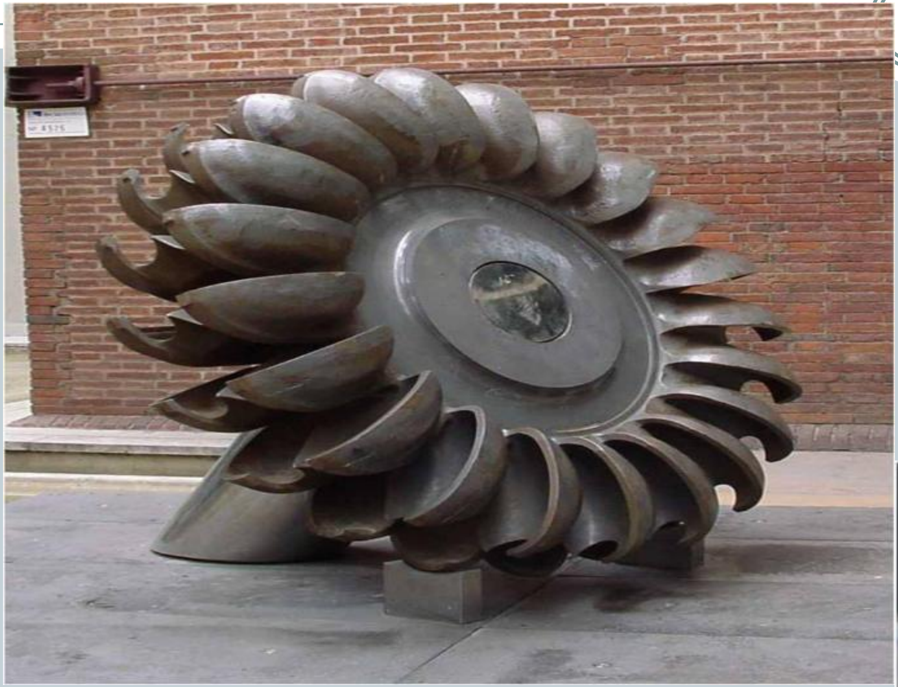
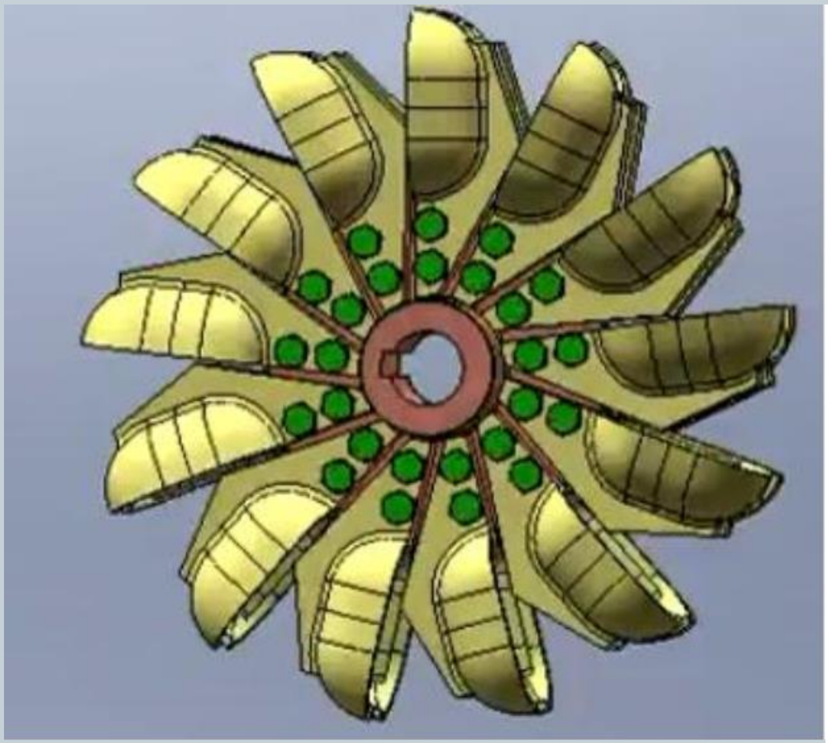


Fig. 2.33: VIEW SHOWING THE SHAPE OF THE BUCKET

Hydraulic Turbines



Hydraulic Turbines



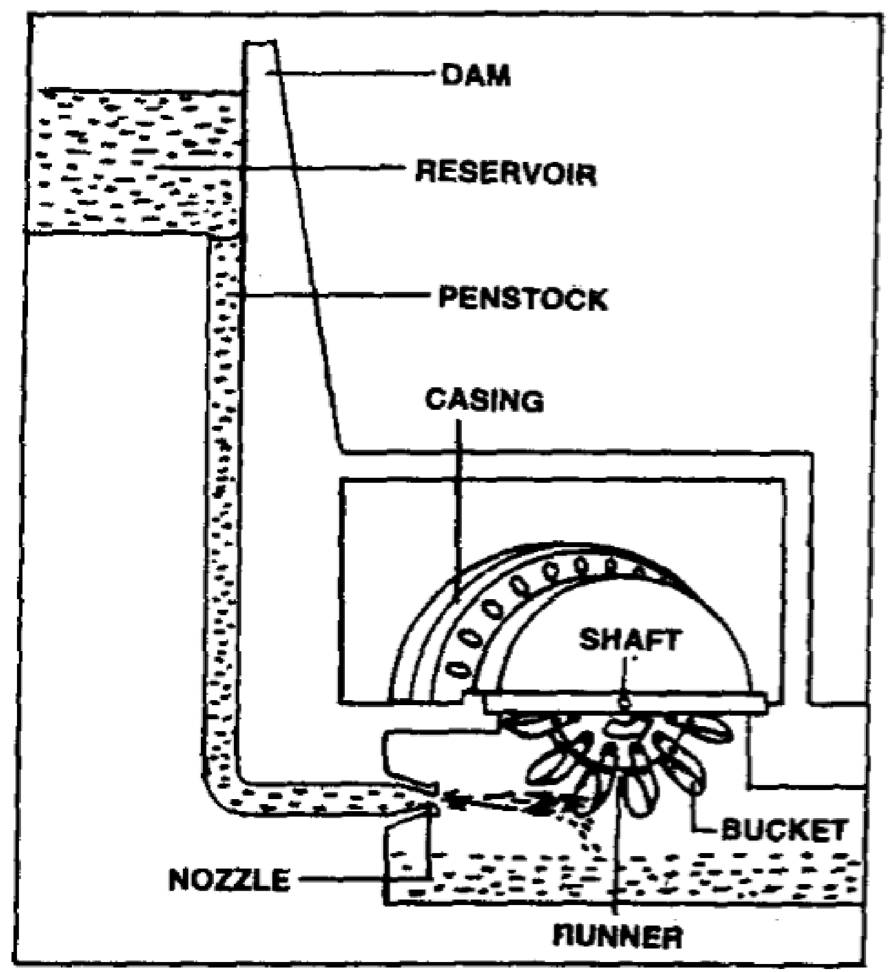


Fig. 2.31: WORKING OF A PELTON WHEEL

Hydraulic Turbines

Kaplan turbine (Propeller turbine):

- Kaplan turbine is a type of propeller turbine which was developed during 1900’s by the Austrian engineer Victor Kaplan (1876 – 1934)
- Kaplan turbine is a low head turbine and used for heads of less than 80 metres.
- The runner of a kaplan turbine resembles with propeller of a ship. That is why, a Kaplan turbine is also called as propeller turbine.

Hydraulic Turbines

- The turbine wheel, which is completely under water, is turned by the pressure of water against its blades.
- Guide vanes regulate the amount of water reaching the wheel.

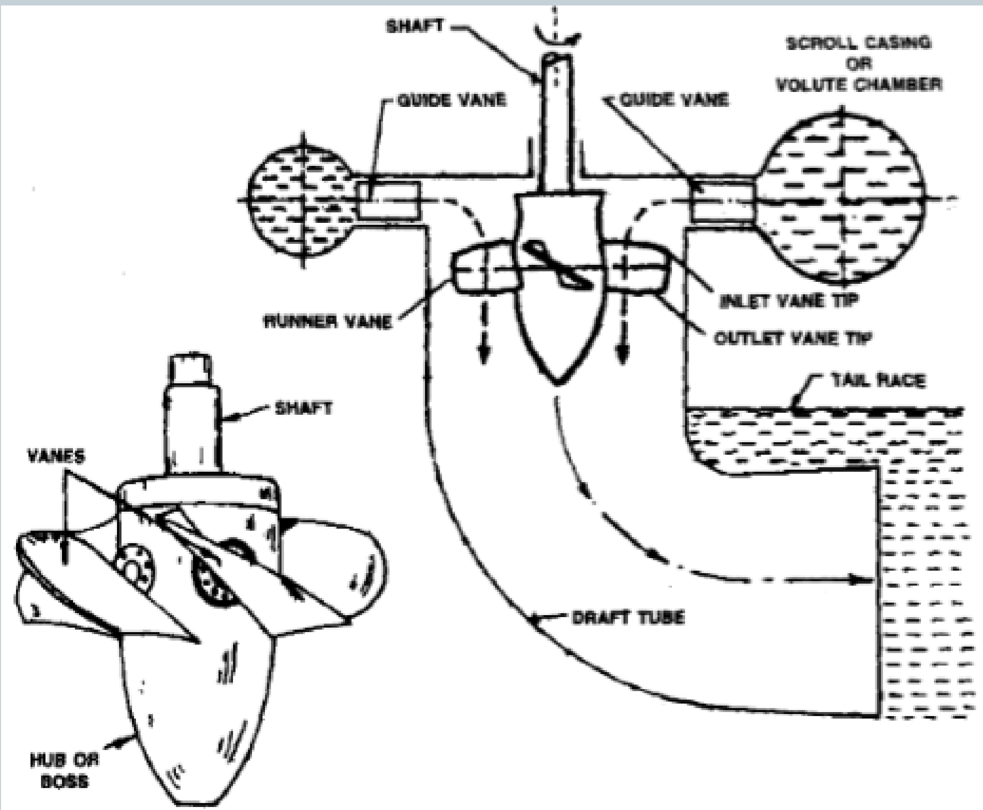


Fig. 2.35: DETAILED VIEW OF THE PROPELLER AND THE KAPLAN TURBINE (elevation view)

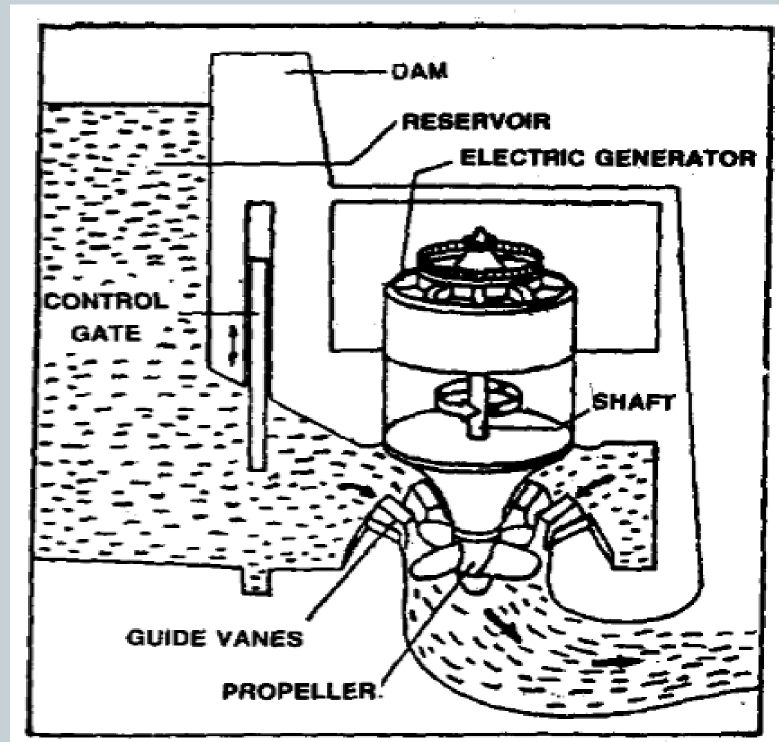
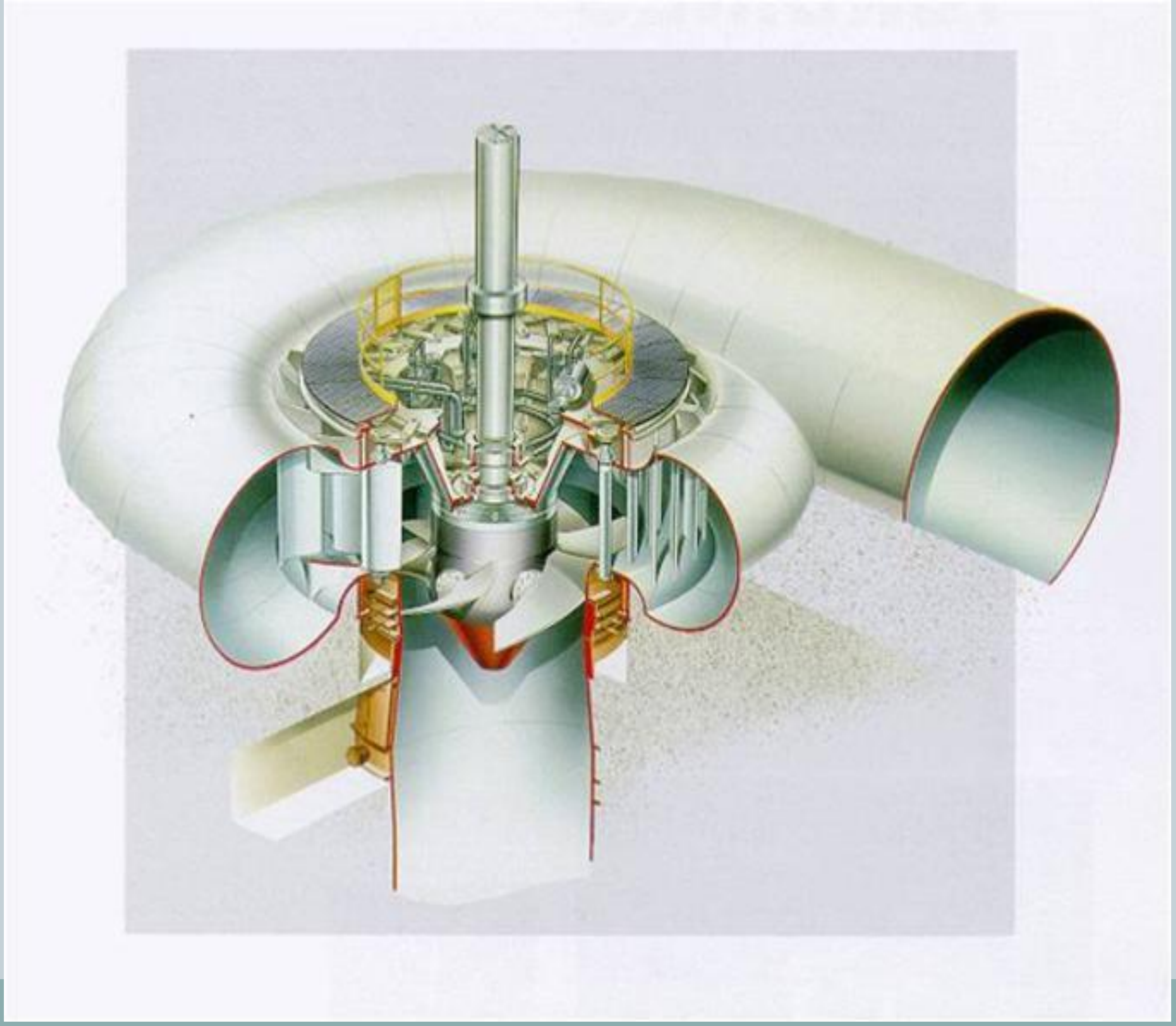


Fig. 2.34: WORKING OF A KAPLAN TURBINE

Hydraulic Turbines



Hydraulic Turbines

Francis turbine :

- A Francis turbine rotates in a closed casing.
- Its wheel has many curved blades called runner vanes as many as 24.
- Its shaft is vertical. The wheel of a Francis turbine operates under water.
- The guide vanes and stay vanes control the amount of water flowing into the runner vanes.
- The runner is rotated mainly due to the weight or pressure of the flowing water.

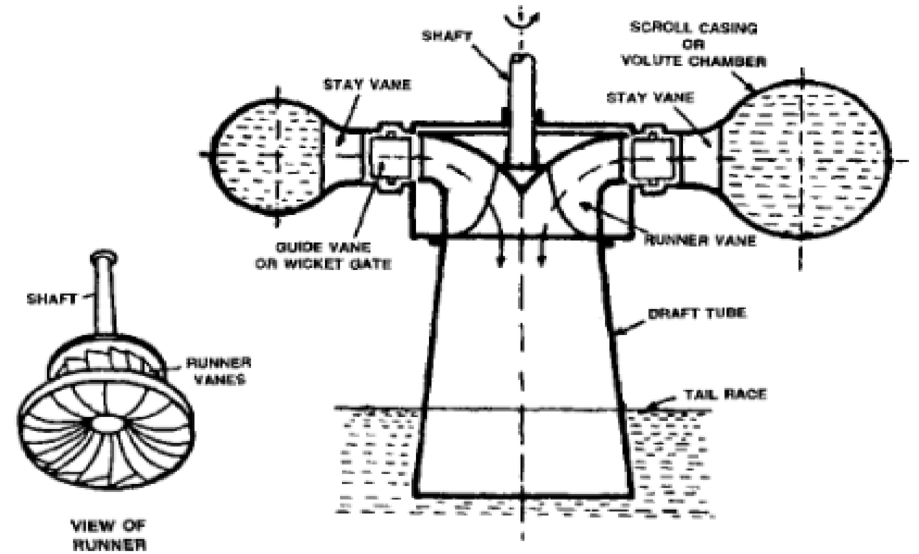
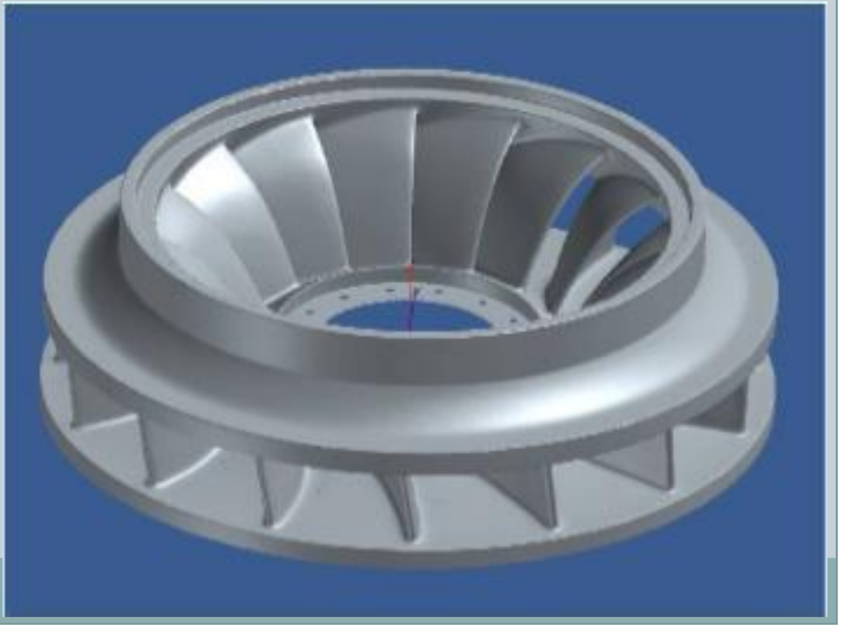
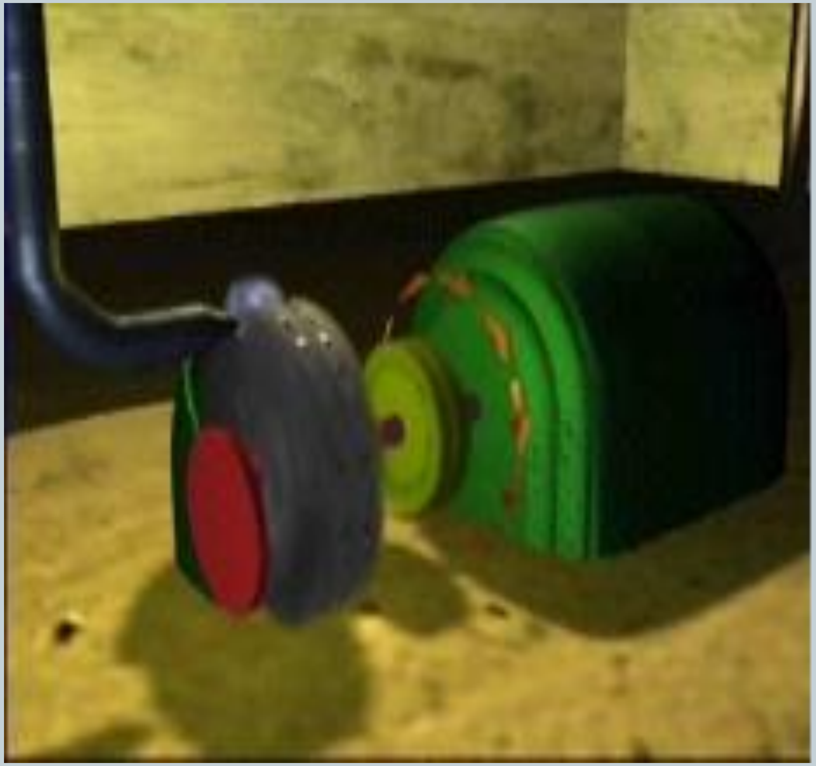


Fig. 2.36: FRANCIS TURBINE (elevation view)

Note : For the purpose of clarity, water is not shown flowing through the vanes and draft tube.



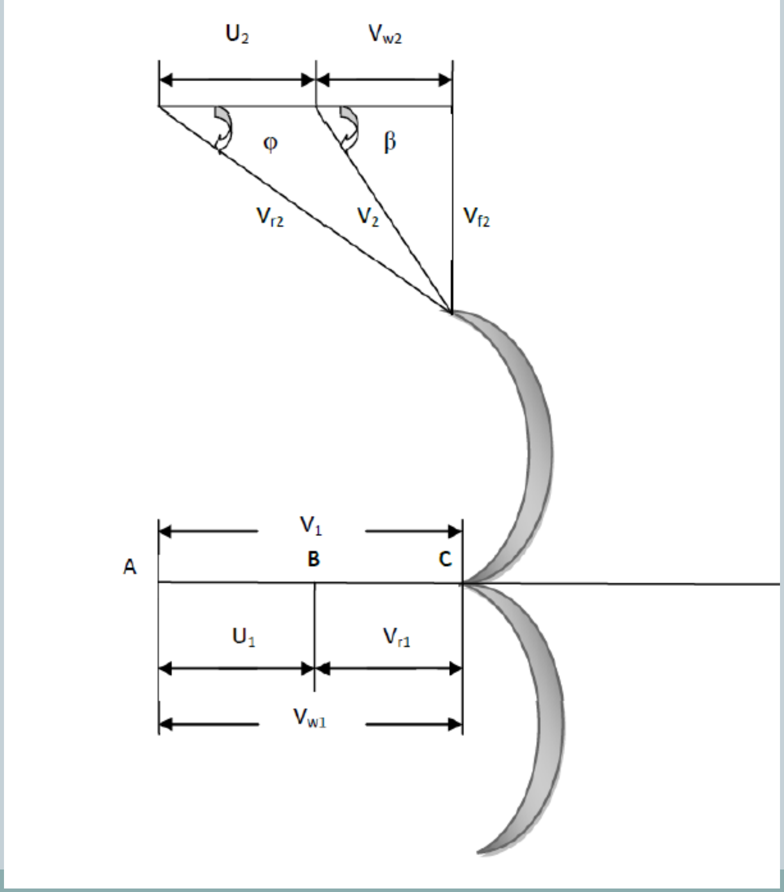
Hydraulic Turbines

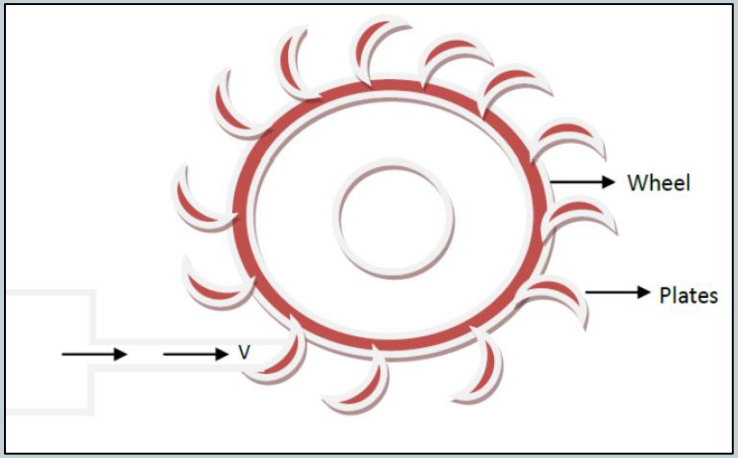


Hydraulic Turbines

Power and efficiency of Pelton turbine:

This is similar to the force exerted by jet of water on the series of curved plates





Hydraulic Turbines

The velocity Δ at inlet will be a straight line,

$$V_{w1} = V_1$$

$$V_{r1} = V_1 - u_1 = V_1 - u$$

The velocity Δ at outlet,

$$V_{r1} = V_{r2}, \qquad V_{w2} = V_{r2}\cos\emptyset - u_2$$

The force exerted by jet of water in the direction of motion,

$$F_x = \rho a V_1 [V_{w1} + V_{w2}]$$

Mass of water striking $= \rho a V_1$

Workdone by the jet on the runner / sec $= F_x \text{ x } u$ $= \rho a V_1 [V_{w1} + V_{w2}] \text{x } u, \quad Nm/s$

Power given to the runner by jet $= \dfrac{\rho a V_1 [V_{w1} + V_{w2}] \text{x } u}{1000}$ $, KW$

Hydraulic Turbines

$$\text{Workdone/s per unit weight of water} = \frac{\rho a V_1 [V_{w1} + V_{w2}] x\, u}{Weight\ of\ water\ striking/s}$$

$$= \frac{\rho a V_1 [V_{w1} + V_{w2}] x\, u}{\rho a V_1\, x\, g}$$

$$= \frac{[V_{w1} + V_{w2}] x\, u}{g}$$

$$\text{Hydraulic efficiency, } \eta_h = \frac{Workdone/s}{K.E\ of\ jet/s}$$

$$K.E\ of\ jet/s = \frac{1}{2} m V^2$$

$$= \frac{1}{2} \rho a V_1 (V_1{}^2)$$

$$= \frac{1}{2} \rho a V_1{}^3$$

Hydraulic Turbines

$$\text{Hydraulic efficiency, } \eta_h = \frac{\rho a V_1 [V_{w1} + V_{w2}] \text{x u}}{\frac{1}{2}\rho a V_1^{\,3}}$$

$$\eta_h = \frac{2[V_{w1} + V_{w2}] \text{x u}}{V_1^{\,2}}$$

$$\eta_h = \frac{[V_1 + \ (V_1 - u)\cos\emptyset - \text{u}]\text{ x u}}{V_1^{\,2}}$$

$$\eta_h = \frac{2(V_1 - u)\ (1 + \cos\emptyset)\text{x u}}{V_1^{\,2}}$$

The efficiency is maximum when $\frac{d\ (\eta_h)}{du} = 0$

$$\frac{d}{du}\left[\frac{2(V_1 - u)\ (1 + \cos\emptyset)\text{x u}}{V_1^{\,2}}\right] = 0$$

$$\frac{(1 + \cos\emptyset)}{V_1^{\,2}}\ \frac{d}{du}[2u(V_1 - u] = 0$$

Hydraulic Turbines

$$\frac{d}{du}[2uV_1 - 2u^2] = 0$$

$$u = \frac{V_1}{2}$$

Expression for maximum efficiency of pelton wheel

$$\eta_{h_{max}} = \frac{2(V_1 - \frac{V_1}{2})\,(1 + \cos \emptyset)\mathrm{x}\,\frac{V_1}{2}}{{V_1}^2}$$

$$\eta_{h_{max}} = \frac{(1 + \cos \emptyset)}{2}$$

Chapter 7: Reciprocating pump

- Pumps are used to increase the energy level of water by virtue of which it can be raised to a higher level.
- Reciprocating pumps are positive displacement pump, i.e. initially, a small quantity of liquid is taken into a chamber and is physically displaced and forced out with pressure by a moving mechanical elements.
- The use of reciprocating pumps is being limited these days and being replaced by centrifugal pumps.
- For industrial purposes, they have become obsolete due to their high initial and maintenance costs as compared to centrifugal pumps.
- Small hand operated pumps are still in use that include well pumps, etc.
- These are also useful where high heads are required with small discharge, as oil drilling operations.

Reciprocating pump

Main components

- A reciprocation pumps consists of a plunger or a piston that moves forward and backward inside a cylinder with the help of a connecting rod and a crank. The crank is rotated by an external source of power.
- The cylinder is connected to the sump by a suction pipe and to the delivery tank by a delivery pipe.
- At the cylinder ends of these pipes, non-return valves are provided. A non-return valve allows the liquid to pass in only one direction.
- Through suction valve, liquid can only be admitted into the cylinder and through the delivery valve, liquid can only be discharged into the delivery pipe.

Reciprocating pump

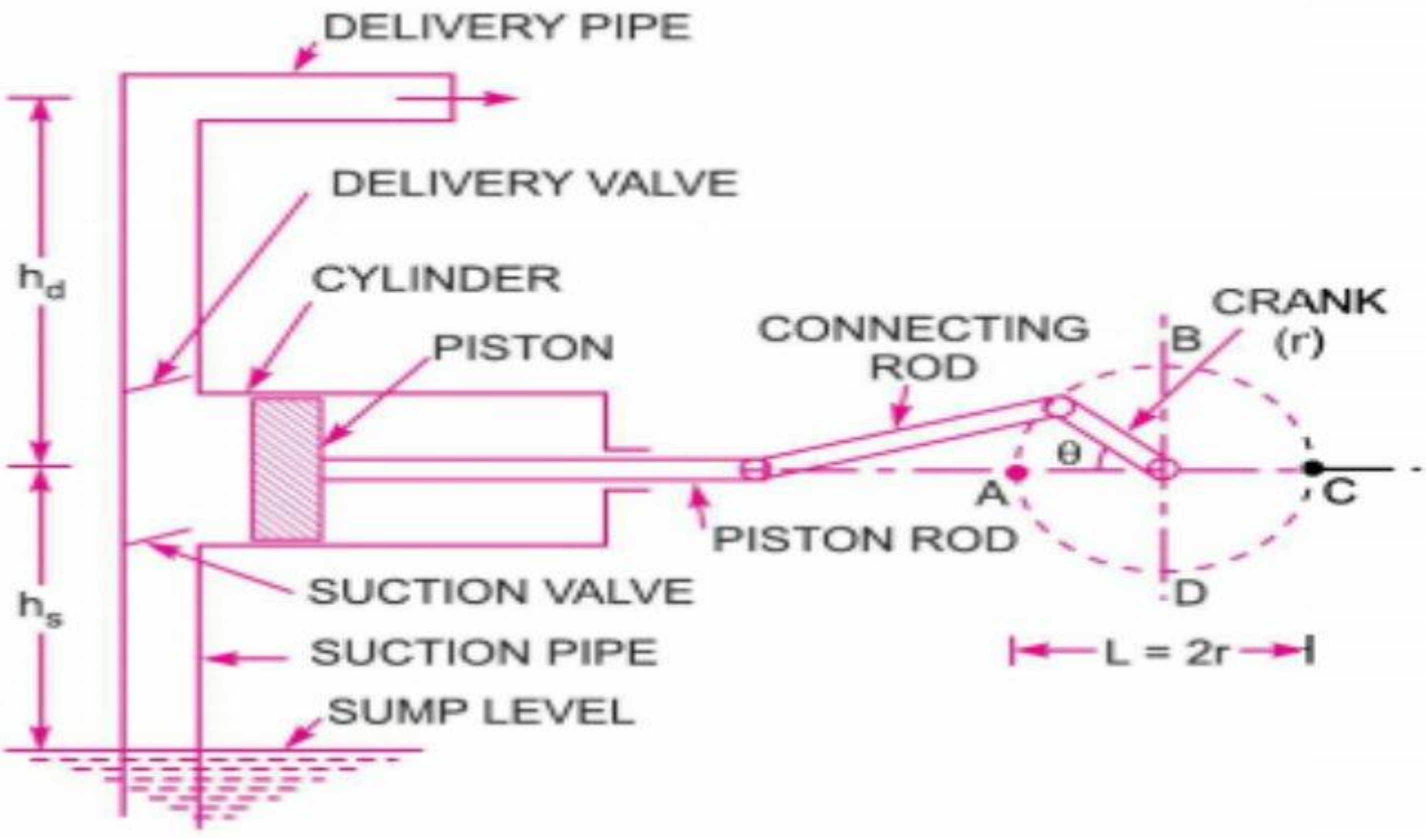


Fig. 20.1 *Main parts of a reciprocating pump.*

Reciprocating pump

Working of Reciprocating Pump:

- When the piston moves from the left to the right, a suction pressure is produced in the cylinder. If the pump is started for the first time or after a long period, air from the suction pipe is sucked during the suction stroke, while the delivery valve is closed. Liquid rises into the suction pipe by a small height due to atmospheric pressure on the sump liquid.
- During the delivery stroke, air in the cylinder is pushed out into the delivery pipe by the thrust of the piston, while the suction valve is closed. When all the air from the suction pipe has been exhausted, the liquid from the sump is able to rise and enter the cylinder.

Reciprocating pump

- During the delivery stroke it is displaced into the delivery pipe. Thus the liquid is delivered into the delivery tank intermittently, i.e. during the delivery stroke only.

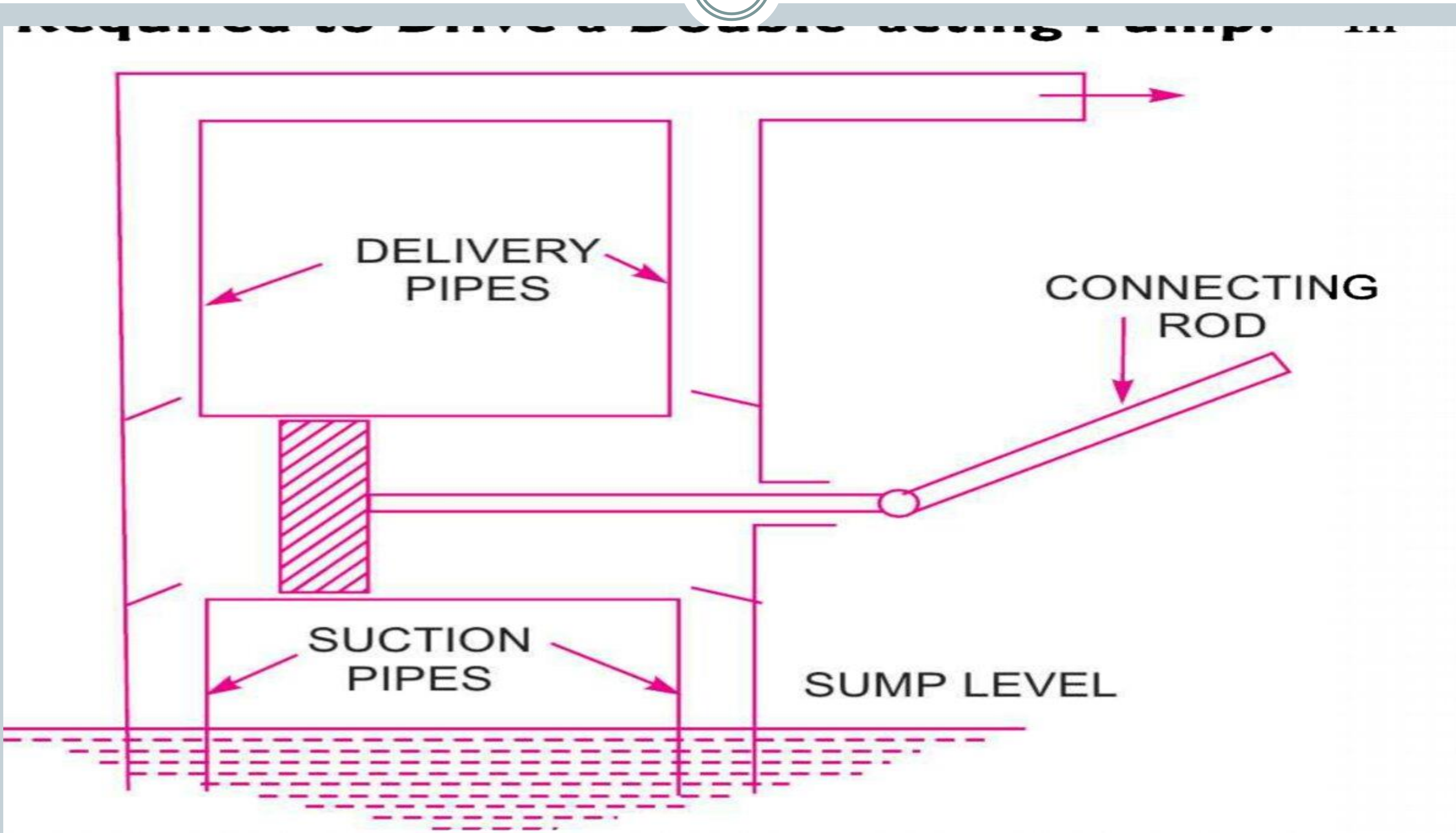
Following are the main types of reciprocating pumps:

- According to use of piston sides
  - Single acting Reciprocating Pump:

    If there is only one suction and one delivery pipe and the liquid is filled only on one side of the piston, it is called a single-acting reciprocating pump.
  - Double acting Reciprocating Pump:

    A double-acting reciprocating pump has two suction and two delivery pipes, Liquid is receiving on both sides of the piston in the cylinder and is delivered into the respective delivery pipes.

Reciprocating pump



Reciprocating pump

- According to number of cylinder
  - Triple cylinder pump (three throw pump)

    A triple-cylinder pump or three throw pump has three cylinders, the cranks of which are set at 120^0 to one another. Each cylinder is provided with its own suction pipe delivery pipe and piston.
  - There can be four-cylinder and five cylinder pumps also, the cranks of which are arranged accordingly.
  - Triple cylinder pump (three throw pump)

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  - There can be four-cylinder and five cylinder pumps also, the cranks of which are arranged accordingly

Reciprocating pump

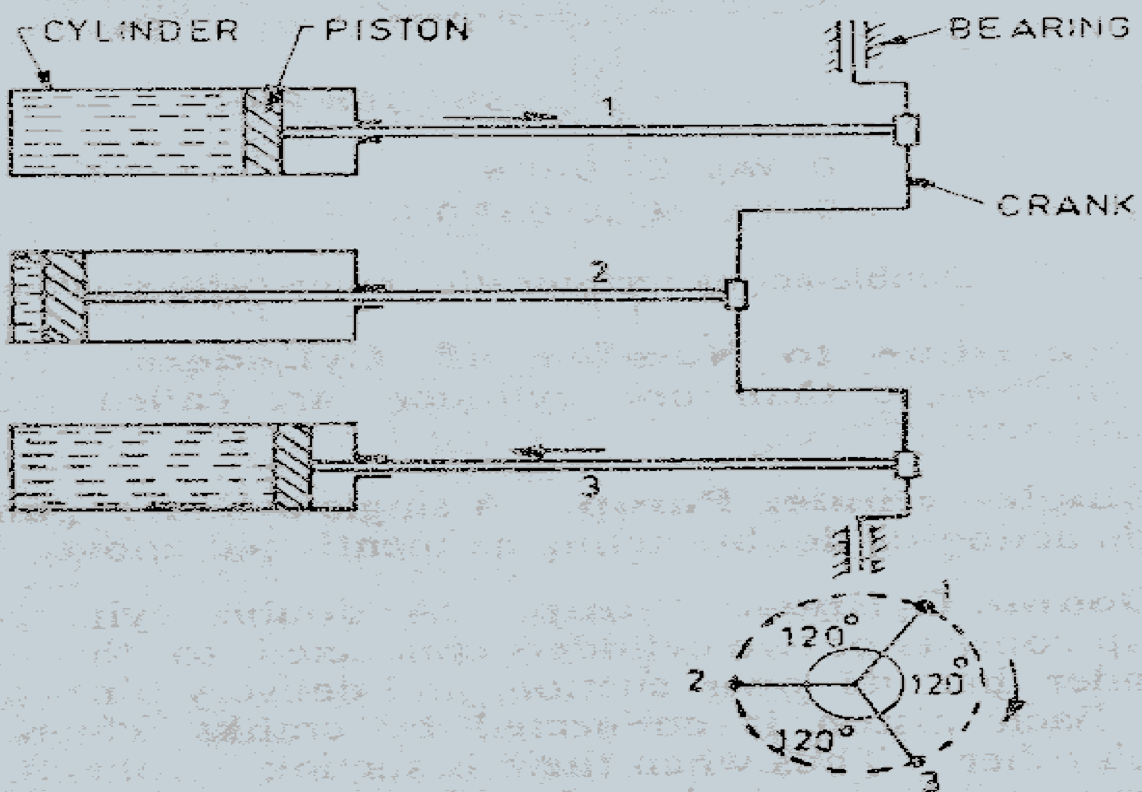


Fig. 11·5. Triple-cylinder pump.

Reciprocating pump

Discharge through a Reciprocating Pump

Let

A = cross sectional area of cylinder

r = crank radius

N = rpm of the crank

L = stroke length (2r)

Discharge through pump per second= Area x stroke length x rpm/60

$$Q_{th} = A \times L \times \frac{N}{60}$$

This will be the discharge when the pump is single acting.

Reciprocating pump

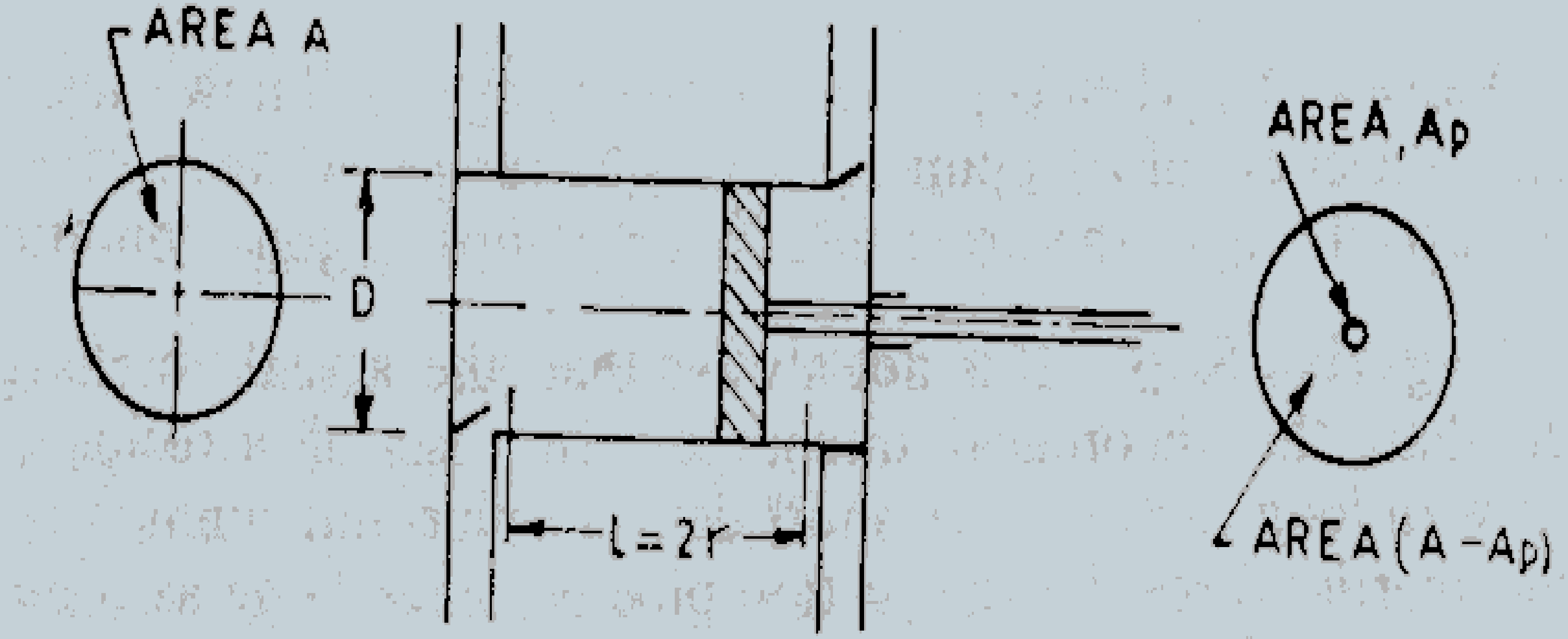


Fig. 11·6. Area of cylinder.

Reciprocating pump

- Discharge in case of double acting pump

Discharge/Second = $$Q_{th} = \left[\frac{ALN}{60} + \frac{(A - A_P)LN}{60} \right]$$

$$Q_{th} = \frac{(2A - A_P)LN}{60}$$

Where, A_p = Area of cross-section of piston rod

However, if area of the piston rod is neglected

Discharge/Second = $$\frac{2ALN}{60}$$

Reciprocating pump

- Thus discharge of a double-acting reciprocating pump is twice than that of a single-acting pump.
- Owing to leakage losses and time delay in closing the valves, actual discharge Q_a usually lesser than the theoretical discharge Q_{th}.

Reciprocating pump

Slip:

Slip of a reciprocating pump is defined as the difference between the theoretical and the actual discharge.

i.e. Slip = Theoretical discharge - Actual discharge

$= Q_{th.} - Q_a$

Slip can also be expressed in terms of %age and given by

$$\%slip = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100$$

$$= \left(1 - \frac{Q_{act}}{Q_{th}}\right)100 = (1 - C_d)100$$

Reciprocating pump

Slip Where C_d is known as co-efficient of discharge and is defined as the ratio of the actual discharge to the theoretical discharge.

$$C_d = Q_a / Q_{th}.$$

Value of C_d when expressed in percentage is known as volumetric efficiency of the pump. Its value ranges between 95---98 %. Percentage slip is of the order of 2% for pumps in good conditions.

Reciprocating pump

Negative slip

- It is not always that the actual discharge is lesser than the theoretical discharge. In case of a reciprocating pump with long suction pipe, short delivery pipe and running at high speed, inertia force in the suction pipe becomes large as compared to the pressure force on the outside of delivery valve. This opens the delivery valve even before the piston has completed its suction stroke. Thus some of the water is pushed into the delivery pipe before the delivery stroke is actually commenced. This way the actual discharge becomes more than the theoretical discharge.

- Thus co-efficient of discharge increases from one and the slip becomes negative.

Reciprocating pump

Power Input

Consider a single acting reciprocating pump.

Let

h_s = Suction head or difference in level between centre line of cylinder and the sump.

h_d = Delivery head or difference in between centre line of cylinder and the outlet of delivery pipe.

H_{st} = Total static head

$= h_s + h_d$

Theoretical work done by the pump

$= \rho\, Q_{th}\, g\, H_{st}$

$$= \rho\left(\frac{ALN}{60}\right)g\left(h_s + h_d\right)$$

Reciprocating pump

Power input to the pump

$$= \rho\left(\frac{ALN}{60}\right)g\left(h_s + h_d\right)$$

However, due to the leakage and frictional losses, actual power input will be more than the theoretical power.

Let η = Efficiency of the pump.

Then actual power input to the pump

$$= \frac{1}{\eta}\rho\left(\frac{ALN}{60}\right)g\left(h_s + h_d\right)$$

Reciprocating pump

Problem-1: A single-acting reciprocating pump discharge 0.018 m^3/s of water per second when running at 60 rpm. Stroke length is 50 cm and the diameter of the piston is 22 cm. If the total lift is 15 m, determine:

a) Theoretical discharge of the pump

b) Slip and percentage slip of the pump

c) Co-efficient of discharge

d) Power required running the pump

Solution:

$L = 0.5$ m

$Q_a = 0.018 m^3/s$

$D = 0.22$ m

$N = 60$ rpm

$H_{st} = 15$ m

Reciprocating pump

Problem-1

Solution:

(a) $$Q_{th} = A \times L \times \frac{N}{60} = \left(\frac{\pi}{4} D^2\right)\frac{LN}{60}$$

$Q_{th} = (\pi/4) x (0.22)^2 x (0.5 x 60/60)$

$Q_{th} =$ **0.019 m³ /s**

(b) Slip = $Q_{th} - Q_a$

Slip = 0.019 – 0.018

= 0.001 m³ /s

Percentage slip = $(Q_{th} - Q_a) / Q_{th}$

= (0.019-0.018)/0.019

= 0.0526 or 5.26%

.

Reciprocating pump

Solution:

(c) $C_d = Q_a / Q_{th}$

$= 0.018/0.019$

= 0.947

(d) Power Input

$= \rho\, Q_{th}\, g\, H_{st}$ (Neglecting Losses)

$= 1000 \times 0.019 \times 9.81 \times 15$

= 2796 w or 2.796 kW

•

Reciprocating pump

Problem-2: A three-throw reciprocating pump delivering 0.1 **m^3 /s** of water against a head of 100 m. Diameter and stroke length of the cylinder are 25 cm and 50 cm respectively. Friction losses amount to 1 m in the suction pipe and 16 m in the delivery pipe. If the velocity of water in the delivery pipe is 1.4 m/s, pump efficiency 90% and slip 2%, determine the pump and the power required.

Solution:

H_{st} = 100 m   Qa = 0.1 **m^3 /s**

D = 0.25 m   L = 0.5 m

h_{fs} = 1 m h_{fd} = 16 m

η_h = 0.9   s = 0.02

Vd = 1.4 m/s

$$Q_{th} = \frac{3ALN}{60}$$

Reciprocating pump

Solution:

We know that, $s = (Q_{th} - Q_a) / Q_{th}$

$0.02 = 1 - Q_a / Q_{th}$

$Q_a / Q_{th} = 0.98$

$Q_{th} = Q_a / 0.98$

$Q_a / 0.98 = 3/60 \times \pi/4 \; D^2 \times L \times N$

$0.1 / 0.98 = 3/60 \times \pi/4 \; (0.25)^2 \times 0.5 \times N$

$\mathbf{N = 83.15\ rpm}$

Total head generated

$H = H_{st} + h_{fs} + h_{fd} + Vd^2/(2g)$

$H = 100+1+16+ (1.4)^2/(2 \times 9.81)$

$H = 117.1\ m$

Reciprocating pump

Solution:

Power required $= 1/\eta_h \, (\rho \, Q_{th} \, g \, H)$

$= 1/0.9 \, (1000 \times 0.1/0.98 \times 9.81 \times 117.1)$

$= 130.21 \times 10^3$ W

= 130.21 KW

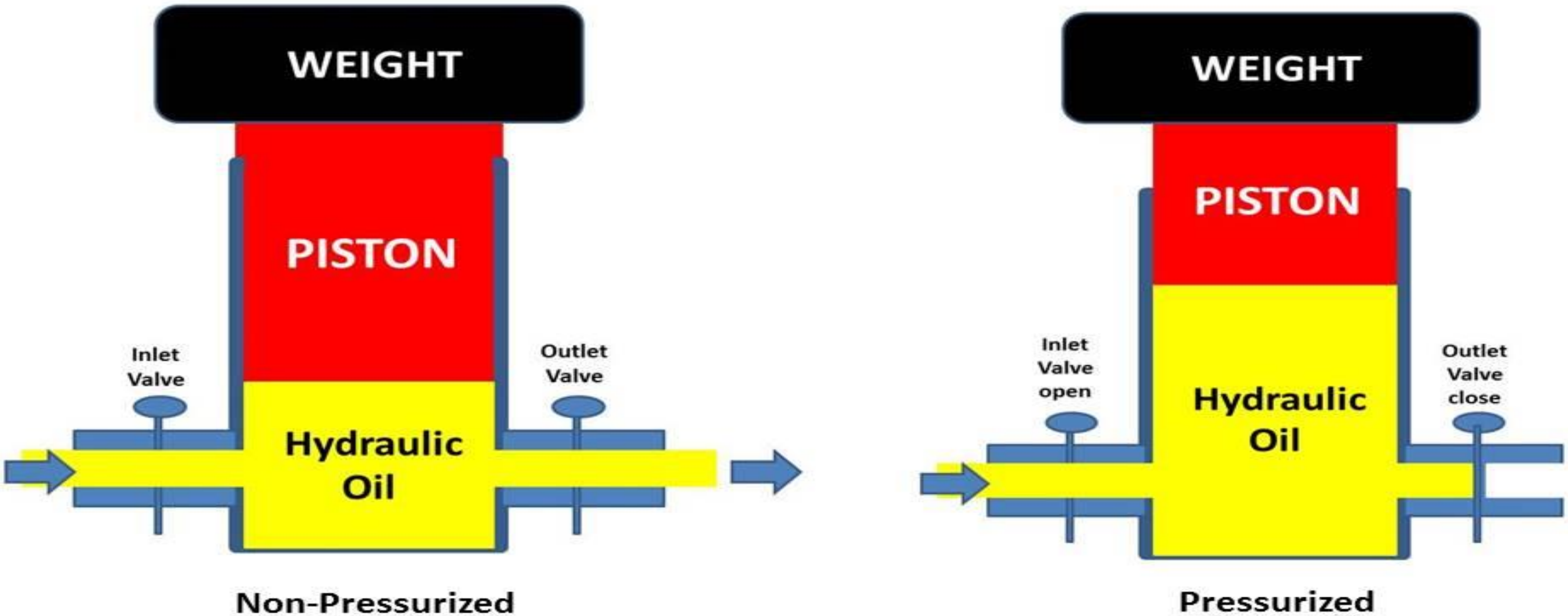
Centrifugal Pumps	Reciprocating Pumps
1. Steady and even flow	**1**. Intermittent and pulsating flow
2. For large discharge, small heads	**2**. For small discharge, high heads.
3. Can be used for viscous fluids e.g. oils, muddy water.	**3**. Can handle pure water or less viscous liquids only otherwise valves give frequent trouble.
4. Low initial cost	**4**. High initial cost.
5. Can run at high speed. Can be coupled directly to electric motor.	**5**. Low speed. Belt drive necessary.
6. Low maintenance cost. Periodic check up sufficient.	**6**. High maintenance cost. Frequent replacement of parts.
7. Compact less floors required.	**7**. Needs 6-7 times area than for centrifugal pumps.
8. Low head pumps have high efficiency	**8**. Efficiency of low head pumps as low as 40 per cent due to the energy losses.
9. Uniform torque	**9**. Torque not uniform.
10. Simple constructions. Less number of spare parts needed	**10**. Complicated construction. More number of spare parts needed.

Hydraulic Accumulator:

- It is a device used for storing the energy of a liquid in the form of pressure energy, which may be supplied for any sudden or intermittent requirement.
- It can be used in case of hydraulic lift or crane.

- When the ram is at lowermost position, the pump supplies water under pressure and it raises the ram on which a heavy weight is placed.
- When the ram is at the upper most position, the cylinder is full of water and accumulator has stored the maximum amount of pressure energy.
- When the machine requires a large amount of pressure energy the hydraulic accumulator will supply this energy and ram will move in downward direction.

Hydraulic Machines

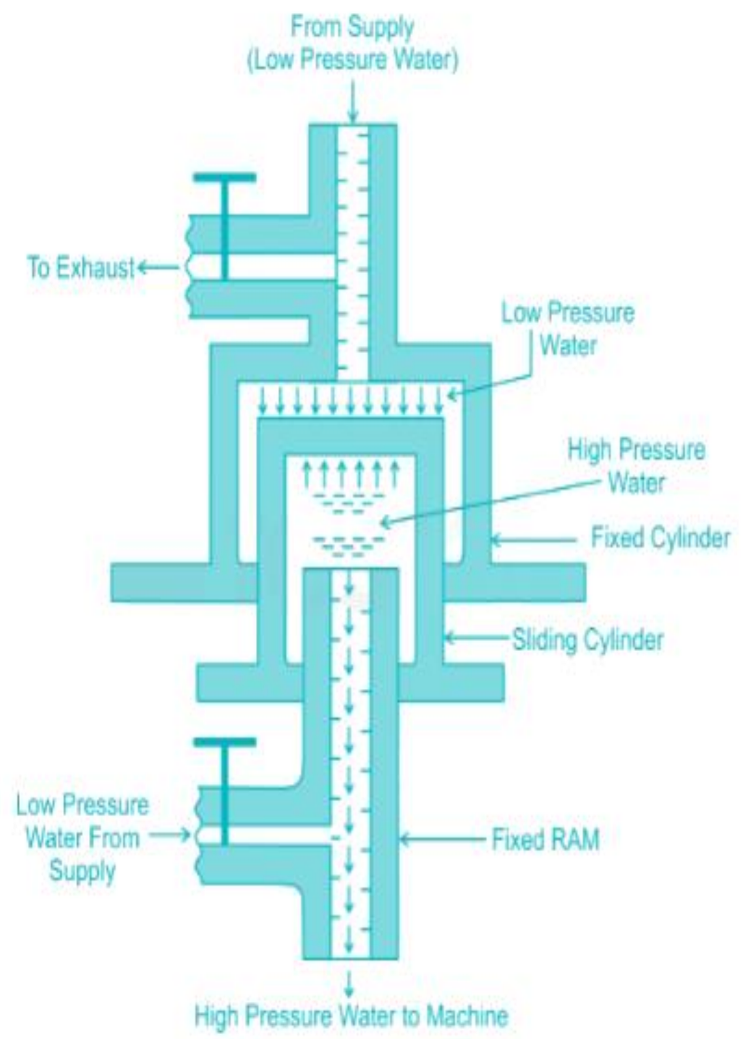


Hydraulic Accumulator

Hydraulic Machines

Hydraulic Intensifier:

- This device is used to increase intensity of pressure of water by means of hydraulic energy available from a large amount of water at a low pressure.
- Such devices are used when hydraulic machines, like hydraulic press requires water at high pressure which cannot be obtained from the main supply directly.
- It consists of a fixed ram through which the water under high pressure, flows to the machine. A hollow inverted cylinder, containing water under high pressure, is mounted over the fixed ram.



Hydraulic Machines

- The inverted sliding cylinder is surrounded by another fixed inverted cylinder which contains water from the main supply at a low pressure.

- A large quantity of water at low pressure from supply enters the inverted fixed cylinder. The weight of this water pressure the sliding cylinder in the downward direction.

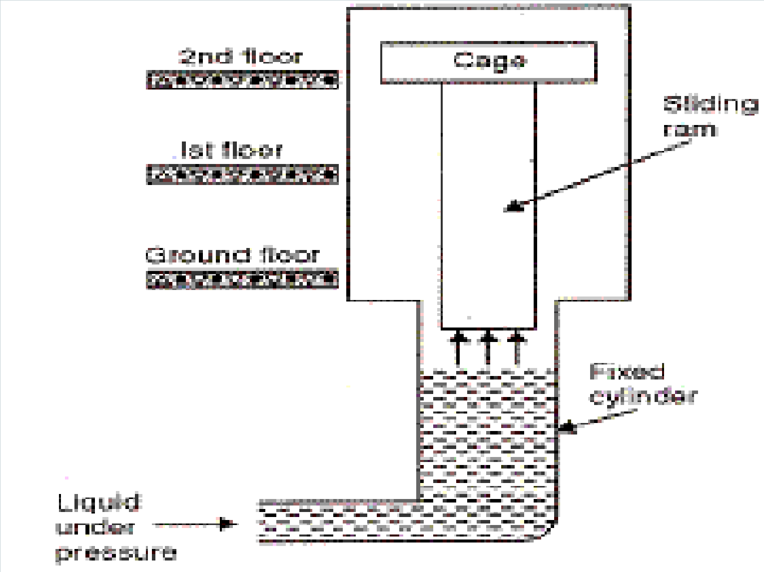
- The water in the sliding cylinder gets compressed due to the downward movement of the sliding cylinder and its pressure is thus increased.

Hydraulic Lift:

It is a device used for carrying passengers or goods from one floor to another in multistoreyed building. It has two types:

1. Direct hydraulic lift
2. Suspended hydraulic lift

1. Direct Hydraulic Lift



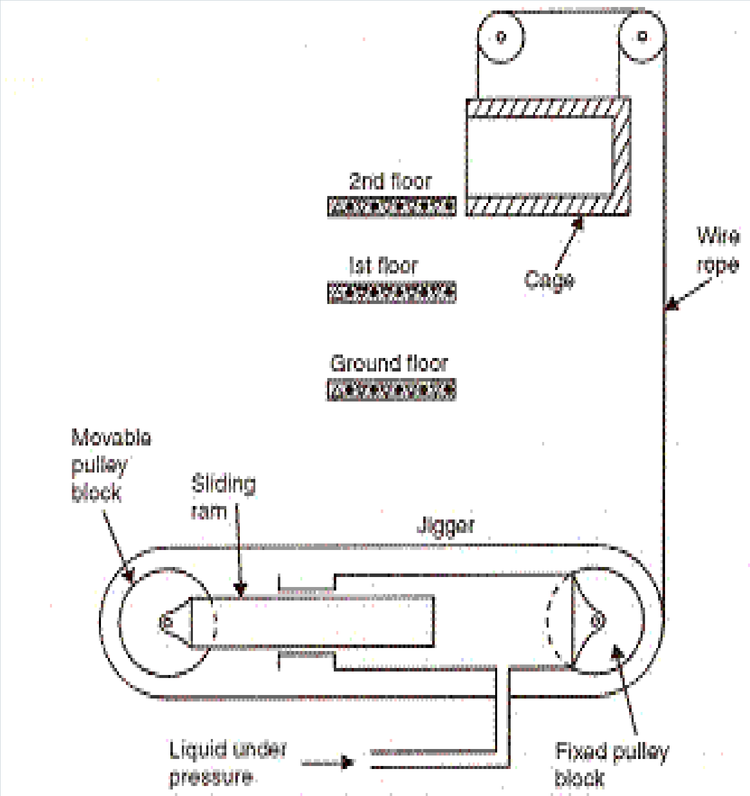
- It consists of a ram, sliding in fixed cylinder. At the top of the sliding ram, a cage is fitted.
- The liquid under pressure flows into the fixed cylinder.
- This liquid exerts force on the sliding ram, which moves vertically up and thus raises the cage to the required height.

- The cage is moved in the downward direction by removing the liquid from the foxed cylinder.

Hydraulic Machines

2. Suspended Hydraulic Lift:

- It is a modified form of direct hydraulic lift. It consists of a cage which is suspended from a wire rope.
- A jigger consisting of a fixed cylinder, a sliding ram and a set of two pulley blocks is provided at the foot of the hole of the cage .
- One of the pulley block is movable while the other one is fixed. The end of the sliding ram is connected to the moving pulley block.
- A wire rope, one end of which is fixed and the other end is taken round all the pulleys of the movable and fixed blocks and finally over the guide lowering of the cage of the lift is done by the jigger.



Hydraulic Machines

Working:

- When water under high pressure is admitted into the fixed cylinder of the jigger, the sliding ram is forced to move towards left.
- As one end of the sliding ram is connected to the movable pulley block, the pulley block moves left, increasing the distance between two pulley blocks.
- The wire rope connected to the cage is pulled and the cage is lifted.
- For lowering the cage, water from the fixed cylinder is taken out.

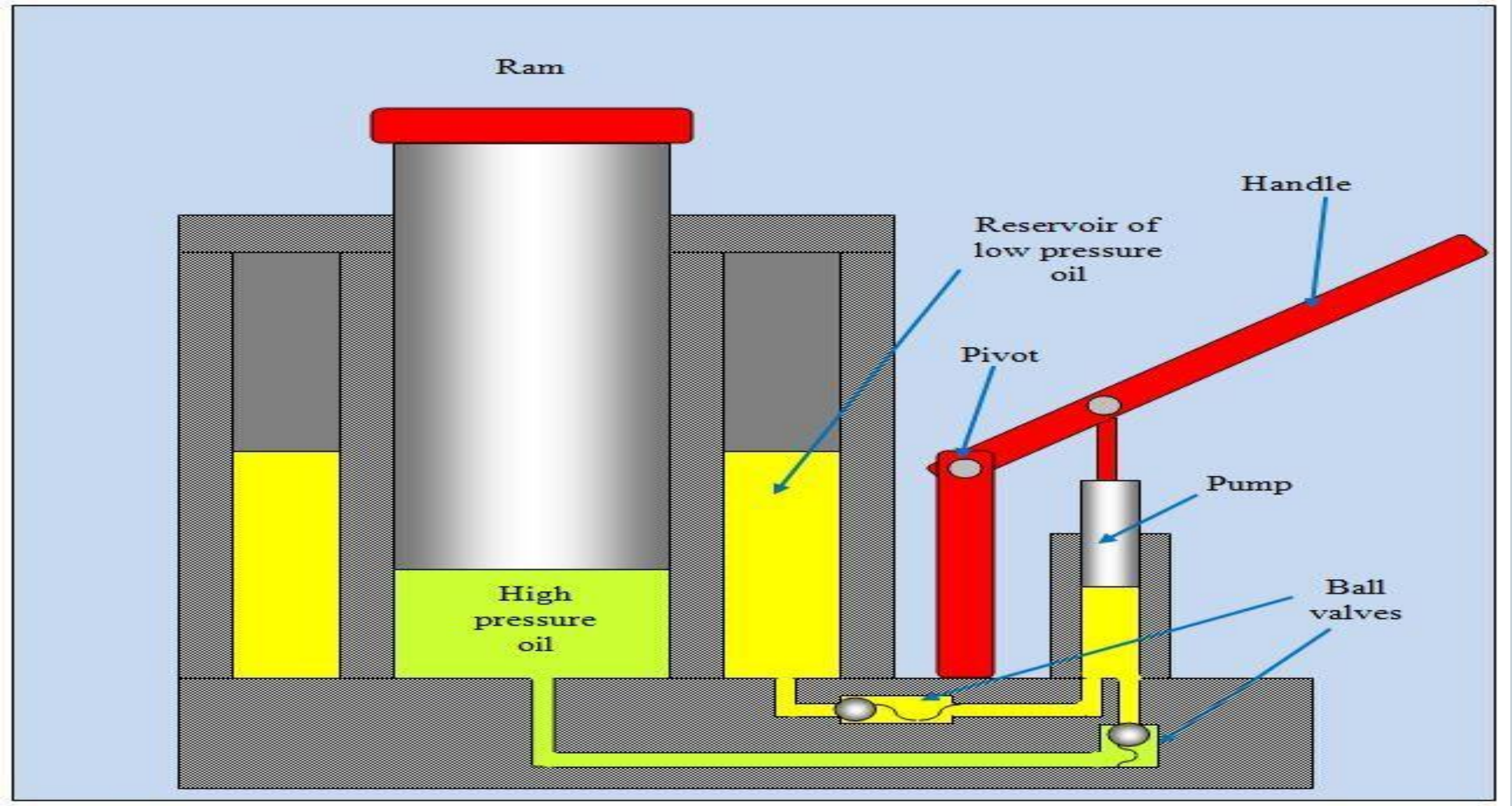
Hydraulic Jack:

- Hydraulic jacks depend on this basic principle to lift heavy loads: they use pump plungers to move oil through two cylinders. The plunger is first drawn back, which opens the suction valve ball within and draws oil into the pump chamber.

- Hydraulic jacks are often used to lift elevators in low and medium rise buildings. A hydraulic jack uses a liquid, which is incompressible, that is forced into a cylinder by a pump plunger. Oil is used since it is self lubricating and stable.

Hydraulic Machines

- Most pieces are made of steel, non steel parts are either rubber or plastic.



Hydraulic Machines

Hydraulic Ram:

- It is a pump which raises water without any external power for its operation.
- When large quantity of water is available at a small height, a small quantity of water can be raised to a greater height with the help of hydraulic ram.
- It works on the principle of water hammer (Hydraulic shock or water hammer; is a pressure surge or wave caused when a fluid, usually a liquid but sometimes also a gas, in motion is forced to stop or change direction suddenly; a momentum change.)

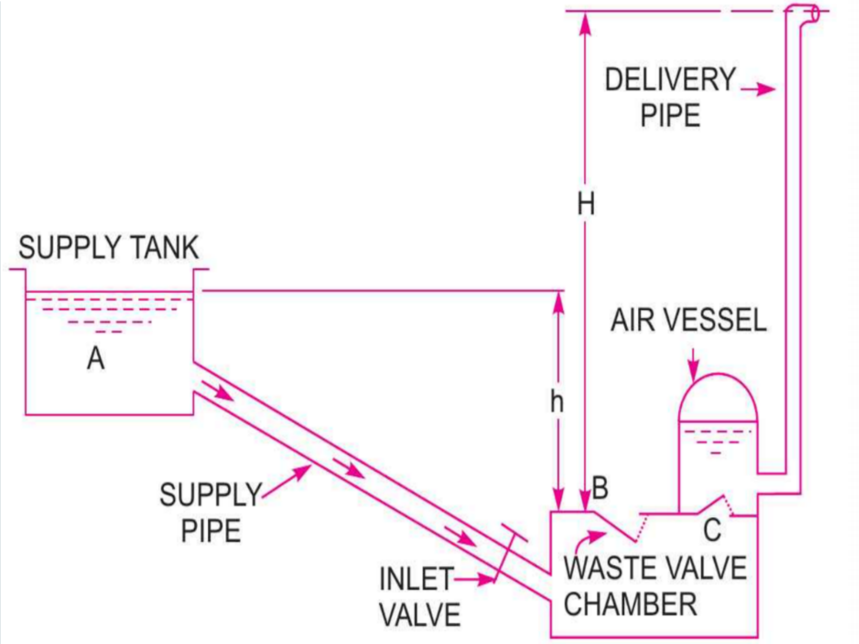


Fig. 21.7 *The hydraulic ram.*

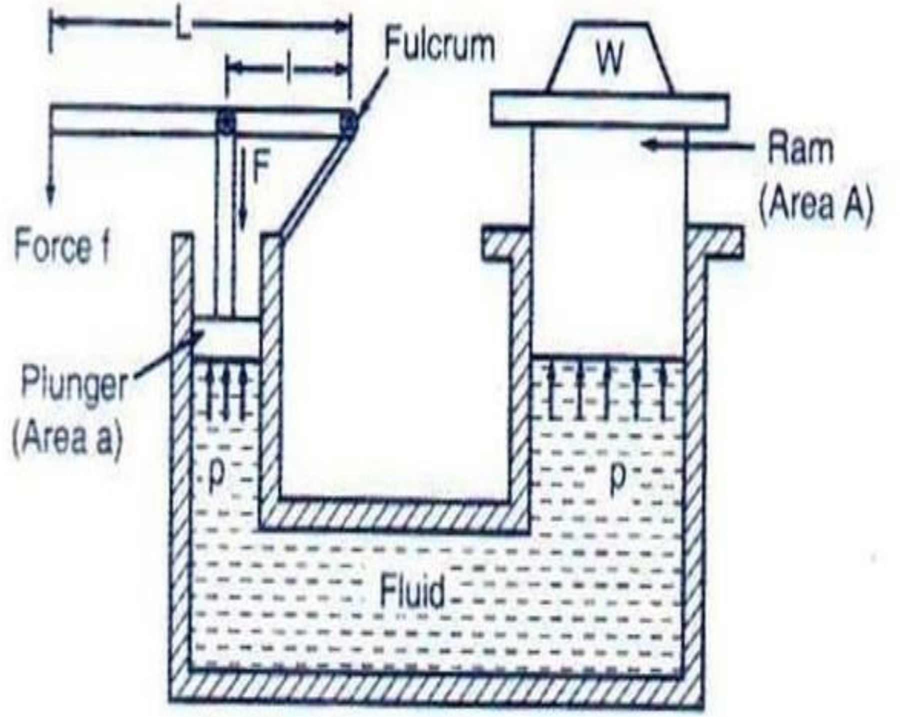
Hydraulic Machines

When the inlet valve fitted to the supply pipe is opened, water starts flowing from the supply tank to the chamber, which has two valves at *B* and *C*. The valve *B* is called waste valve and valve *C* is called the delivery valve. The valve *C* is fitted to an air vessel. As the water is coming into the chamber from supply tank, the level of water rises in the chamber and waste valve *B* starts moving upward. *A* stage comes, when the waste valve *B* suddenly closes. This sudden closure of waste valve creates high pressure inside the chamber. This high pressure force opens the delivery valve *C*. The water from chamber enters the air vessel and compresses the air inside the air vessel. This compressed air exerts force on the water in the air vessel and small quantity of water is raised to a greater height

Hydraulic Machines

Hydraulic Press: It is a device used for lifting heavy weights by the application of a much smaller force.

- It is based on Pascal's law, which states that the intensity of pressure in a static fluid is transmitted equally in all directions.
- It consists if two cylinder of different diameters. One of the cylinders is of large diameter and contains a ram, while the other cylinder is of smaller diameter and contains a plunger.
- The two cylinders are connected by a pipe. The cylinder and pipe contain a liquid through which force is transmitted.



Working principle of hydraulic press

Hydraulic Machines

From the figure:

W = Weight to be lifted

F = Force applied on the plunger.

A = Area of ram,

a = Area of plunger, and

p = Pressure intensity produced by force F.

$$= \frac{\text{Force } F}{\text{Area of plunger}} = \frac{F}{a}$$

Due to Pascal's law, the above intensity of pressure will be equally transmitted in all directions.

Hence, the pressure intensity at the ram will be $= p = \dfrac{F}{a}$.

But the pressure intensity on ram is also $= \dfrac{\text{Weight}}{\text{Area of ram}} = \dfrac{W}{A}$.

Equating the pressure intensity on ram, $\dfrac{F}{a} = \dfrac{W}{A}$

$$\therefore \qquad W = \frac{F}{a} \times A. \qquad \text{...(21.1)}$$

Hydraulic Machines

Problem 21.1 *A hydraulic press has a ram of 300 mm diameter and a plunger of 45 mm diameter. Find the weight lifted by the hydraulic press when the force applied at the plunger is 50 N.*

Solution. Given :

Diameter of ram, $D = 300 \text{ mm} = 0.30 \text{ m}$

Diameter of plunger, $d = 45 \text{ mm} = 0.045 \text{ m}$

Force on plunger, $F = 50 \text{ N}$

Let weight lifted $= W \text{ N}$

Area of ram, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.30)^2 = 0.07068 \text{ m}^2$

Area of plunger, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.045)^2 = .00159 \text{ m}^2$

The weight lifted (W) is given by equation (21.1) as

$$W = \frac{F}{a} \times A = \frac{50 \times .07068}{.00159} = \mathbf{2222.64 \text{ N. Ans.}}$$

Hydraulic Machines

Problem 21.4 *If in the problem 21.1, the stroke of the plunger is 100 mm, find the distance travelled by the weight in 100 strokes. Determine the work done during 100 strokes.*

Solution. The data given in problem 21.1 :

$$D = 0.30 \text{ m}, A = 0.07068 \text{ m}^2, d = 0.045 \text{ m}, a = .00159 \text{ m}^2$$

$$F = 50 \text{ N and } W \text{ (calculated)} = 2222.64 \text{ N}$$

Stroke of plunger $= 100 \text{ mm} = 0.10 \text{ m}$

Number of strokes $= 100$

Volume of liquid displaced by plunger in one stroke

$$= \text{Area of plunger} \times \text{Stroke of plunger}$$

$$= a \times 0.10 \text{ m}^3 = .00159 \times 0.10 = .000159 \text{ m}^3.$$

The liquid displaced by plunger will enter the cylinder in which ram is fitted and this liquid will move the ram in the upward direction.

Let the distance moved by the ram or weight in one stroke

$$= x \text{ m}$$

Then volume displaced by ram in one stroke

$$= \text{Area of ram} \times x = A \times x = 0.07068 \times x \text{ m}^3$$

As volume displaced by plunger and ram is the same,

$$\therefore \quad .000159 = .07068 \times x$$

$$\therefore \quad x = \frac{.000159}{.07068} = .00225 \text{ m}$$

$\therefore$ Distance moved by weight in 100 strokes

$$= x \times 100 = .00225 \times 100 = \mathbf{0.225 \text{ m. Ans.}}$$

Work done during 100 strokes $= \text{Weight lifted} \times \text{Distance moved}$

$$= W \times 0.225 = 2222.64 \times 0.225 \text{ Nm} = \mathbf{500.094 \text{ Nm. Ans.}}$$

Hydraulic Machines

Hydraulic Crane:

- It is a device use for raising or transferring loads. It is widely used in workshops, warehouses and dock sidings.
- It consists of a mast, tie, jib, guide pulley and a jigger.
- The jib and tie are attached to the mast. The jib can be raised or lowered in order to decrease or increase the radius of action of the crane.
- The mast along with the jib can revolve about a vertical axis and thus the load attached to the rope can be transferred to any place within the area of the crane's action.
- The jigger which consists of a movable ram sliding in a fixed cylinder, is used for lifting or lowering the heavy loads.
- One ened of the ram is in contact with water and the other is connected to set of movable pulley block.
- Another pulley block called fixed pulley block is attached to the fixed cylinder.
- The pulley block, attached to the ram, moves up and down while the pulley block, attached to the fixed cylinder, is not having any movement.

Hydraulic Machines

- A wire rope, with one end fixed to movable pulley is taken around all the pulleys of the two sets of pulleys and the guide pulley(attached to jib). The other end of the rope os provided with a hook, for suspending loads.
- For lifting the load, water under high pressure is admitted into the cylinder of the jigger. This water forces the sliding ram to move vertically up.
- Due to vertical movement of the ram, the movable pulley also moves up. This increases the distance between the two pulley blocks and the wire passing over the guide pulley is pulled by the jigger. This raises the load attached to the hook.

Hydraulic Machines