Introduction to mechanisms, velocity and acceleration analysis of mechanisms

1.1 Introduction

Mechanics: It is that branch of scientific analysis which deals with motion, time and force.

Kinematics is the study of motion, without considering the forces which produce that motion. Kinematics of machines deals with the study of the relative motion of machine parts. It involves the study of position, displacement, velocity and acceleration of machine parts.

Dynamics of machines involves the study of forces acting on the machine parts and the motions resulting from these forces.

Plane motion: A body has plane motion, if all its points move in planes which are parallel to some reference plane. A body with plane motion will have only three degrees of freedom. I.e., linear along two axes parallel to the reference plane and rotational/angular about the axis perpendicular to the reference plane. (eg. linear along X and Z and rotational about Y.)The reference plane is called plane of motion. Plane motion can be of three types. 1) Translation 2) rotation and 3) combination of translation and rotation.

Kinematic link (or) element

A machine part or a component of a mechanism is called a kinematic link or simply a link. A link is assumed to be completely rigid, or under the action of forces it does not suffer any deformation, signifying that the distance between any two points on it remains constant. Although all real machine parts are flexible to some degree, it is common practice to assume that deflections are negligible and parts are rigid when analyzing a machine’s kinematic performance.

Types of link

(a) Based on number of elements of link:

Binary link: Link which is connected to other links at two points. (Fig.1.3 a)

Ternary link: Link which is connected to other links at three points. (Fig.1.3 b)

Quaternary link: Link which is connected to other links at four points. (Fig1.3 c)

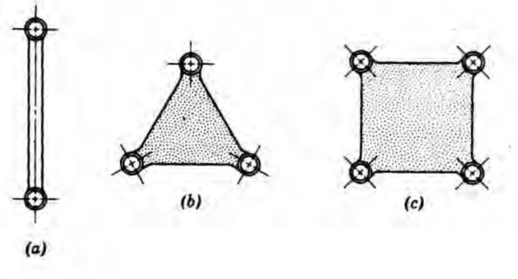


Fig.1.3

(a) Based on type of structural behavior:

Sometimes, a machine member may possess one-way rigidity and is capable of transmitting the force in one direction with negligible deformation. Examples are (a) chains, belts and ropes which are resistant to tensile forces, and (b) fluids which are resistant to compressive forces and are used as links in hydraulic presses, brakes and jacks. In order to transmit motion, the driver and the follower may be connected by the following three types of links:

1. *Rigid link.* A rigid link is one which does not undergo any deformation while transmitting motion. Strictly speaking, rigid links do not exist. However, as the deformation of a connecting rod, crank etc. of a reciprocating steam engine is not appreciable, they can be considered as rigid links.

2. *Flexible link.* A flexible link is one which is partly deformed in a manner not to affect the transmission of motion. For example, belts, ropes, chains and wires are flexible links and transmit tensile forces only.

3. *Fluid link.* A fluid link is one which is formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only, as in the case of hydraulic presses, jacks and brakes.

1.4 Structure

It is an assemblage of a number of resistant bodies (known as members) having no relative motion between them and meant for carrying loads having straining action. A railway bridge, a roof truss, machine frames etc., are the examples of a structure.

Machine: A machine is a mechanism or collection of mechanisms, which transmit force from the source of power to the resistance to be overcome. Though all machines are mechanisms, all mechanisms are not machines. Many instruments are mechanisms but are not machines, because they do no useful work nor do they transform energy.

Difference between structure & machine

The following differences between a machine and a structure are important from the subject point of view:

1. The parts of a machine move relative to one another, whereas the members of a structure do not move relative to one another.

2. A machine transforms the available energy into some useful work, whereas in a structure no energy is transformed into useful work.

3. The links of a machine may transmit both power and motion, while the members of a structure transmits forces only.

Comparison of Mechanism, Machine and Structure

Mechanism	Machine	Structure
1. There is relative motion between the parts of a mechanism	Relative motion exists between parts of a machine.	There is no relative motion between the members of a structure. It is rigid as a whole.
2. A mechanism modifies and transmits motion.	A machine consists of one or more mechanisms and hence transforms motion	A structure does not transform motion.
3. A mechanism does not transmit forces and does not do work	A machine modifies energy or do some work	A structure does not do work. It only transmits forces.
4. Mechanisms are dealt with in kinematics.	Machines are dealt with in kinetics.	Structures are dealt with in statics.

1.6 Kinematic pair

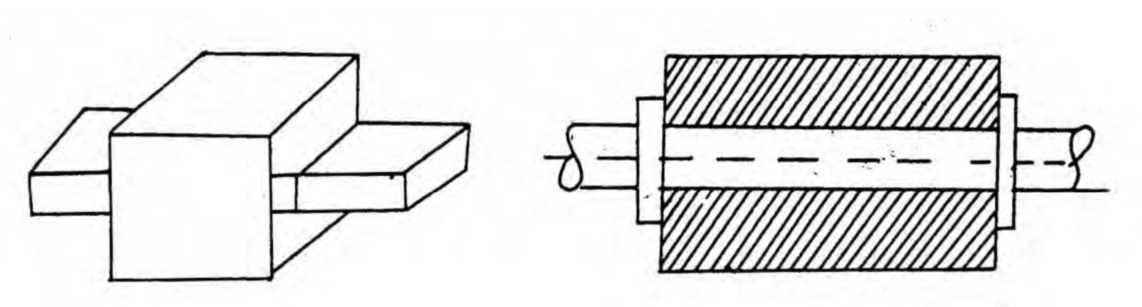
The two links or elements of a machine, when in contact with each other, are said to form a pair. If the relative motion between them is completely or successfully constrained (i.e. in a definite direction), the pair is known as **kinematic pair**.

Classification of kinematic pair

The kinematic pairs may be classified according to the following considerations :

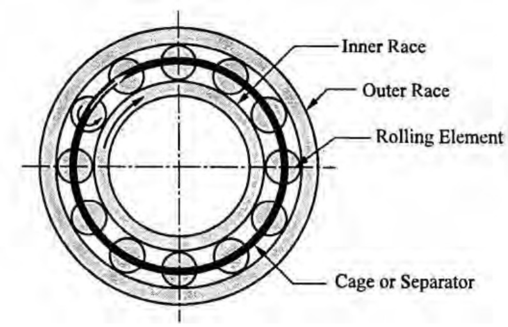
(i) Based on nature of contact between elements:

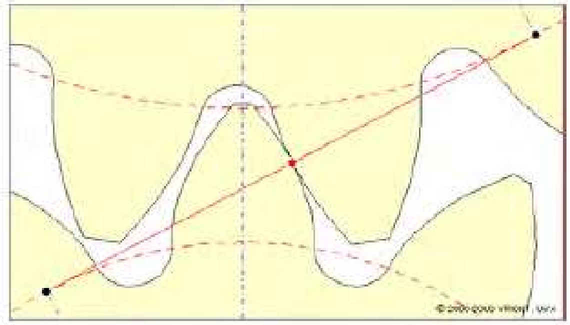
(a) Lower pair. If the joint by which two members are connected has surface contact, the pair is known as lower pair. Eg. pin joints, shaft rotating in bush, slider in slider crank mechanism.



Lower pairs

(b) Higher pair. If the contact between the pairing elements takes place at a point or along a line, such as in a ball bearing or between two gear teeth in contact, it is known as a higher pair.





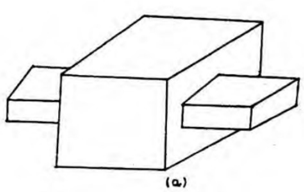
Higher pairs

(ii) Based on relative motion between pairing elements:

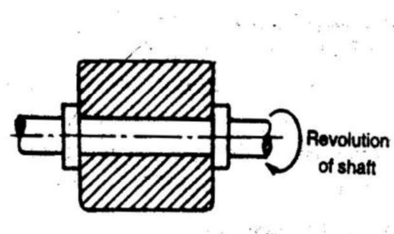
(a) Siding pair. Sliding pair is constituted by two elements so connected that one is constrained to have a sliding motion relative to the other. DOF = 1

(b) Turning pair (revolute pair). When connections of the two elements are such that only a constrained motion of rotation of one element with respect to the other is possible, the pair constitutes a turning pair. DOF = 1

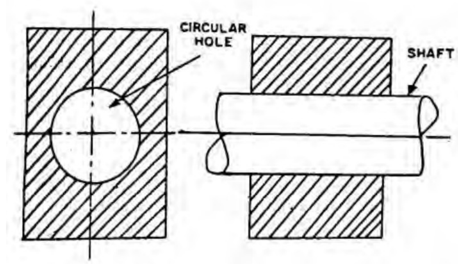
(c) Cylindrical pair. If the relative motion between the pairing elements is the combination of turning and sliding, then it is called as cylindrical pair. DOF = 2



Sliding pair

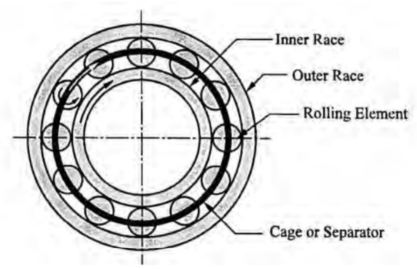


Turning pair

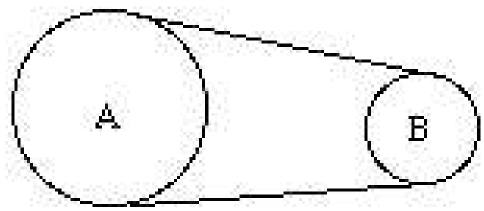


Cylindrical pair

(d) Rolling pair. When the pairing elements have rolling contact, the pair formed is called rolling pair. Eg. Bearings, Belt and pulley. DOF = 1



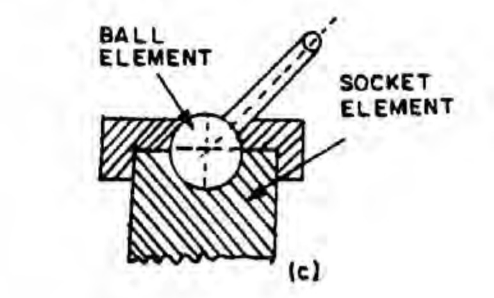
Ball bearing



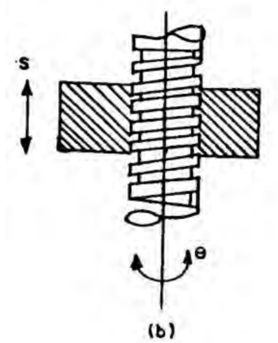
Belt and pulley

(e) Spherical pair. A spherical pair will have surface contact and three degrees of freedom. Eg. Ball and socket joint. DOF = 3

(f) Helical pair or screw pair. When the nature of contact between the elements of a pair is such that one element can turn about the other by screw threads, it is known as screw pair. Eg. Nut and bolt. DOF = 1



Ball and socket joint



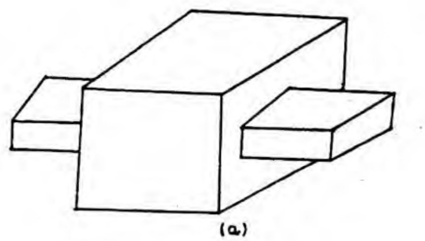
Screw pair

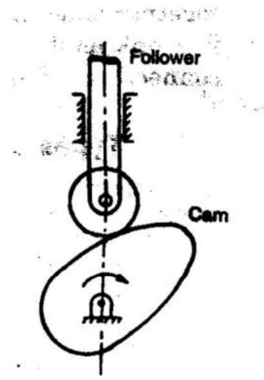
(a) Sliding pair (prismatic pair) eg. piston and cylinder, crosshead and slides, tail stock on lathe bed.
(b) Turning pair (Revolute pair): eg: cycle wheel on axle, lathe spindle in head stock.
(c) Cylindrical pair: eg. shaft turning in journal bearing.
(d) Screw pair (Helical pair): eg. bolt and nut, lead screw of lathe with nut, screw jack.
(e) Spherical pair: eg. penholder on stand, castor balls.

(iii) Based on the nature of mechanical constraint.

(a) Closed pair. Elements of pairs held together mechanically due to their geometry constitute a closed pair. They are also called form-closed or self-closed pair.

(b) Unclosed or force closed pair. Elements of pairs held together by the action of external forces constitute unclosed or force closed pair .Eg. Cam and follower.





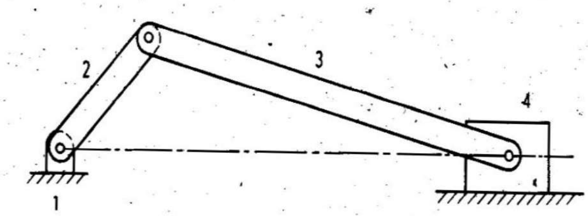
Closed pair

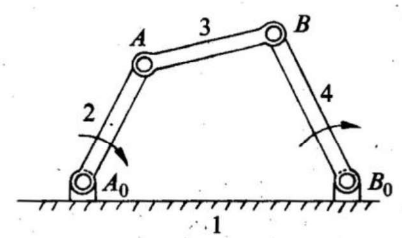
Force closed pair (cam & follower)

Mechanism

When one of the links of a kinematic chain is fixed, the chain is known as ***mechanism***. A mechanism with four links is known as ***simple mechanism***, and the mechanism with more than four links is known as ***compound mechanism***. When a mechanism is required to transmit power or to do some particular type of work, it then becomes a ***machine***.

A mechanism is a constrained kinematic chain. This means that the motion of any one link in the kinematic chain will give a definite and predictable motion relative to each of the others. Usually one of the links of the kinematic chain is fixed in a mechanism.



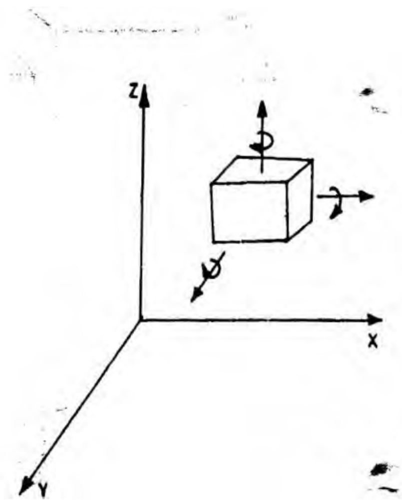


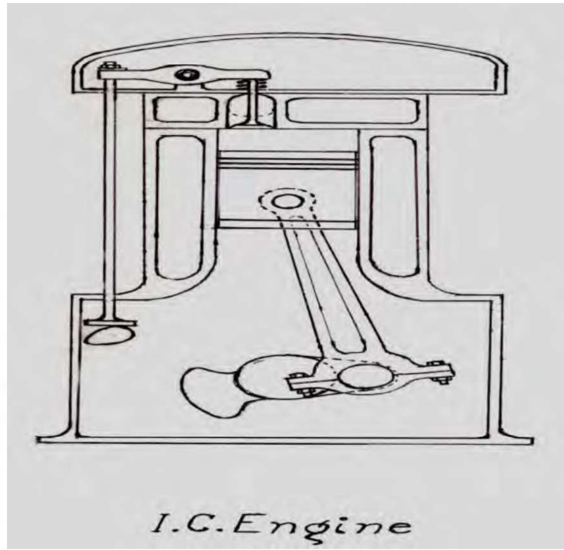
Slider crank and four bar mechanisms.

Number of degrees of freedom for plane mechanism

Degrees of freedom/mobility of a mechanism: The number of independent input parameters (or pair variables) that are needed to determine the position of all the links of the mechanism with respect to the fixed link is termed its degrees of freedom.

Degrees of freedom (DOF) is the number of independent coordinates required to describe the position of a body in space. A free body in space can have six degrees of freedom. I.e., linear positions along x, y and z axes and rotational/angular positions with respect to x, y and z axes. In a kinematic pair, depending on the constraints imposed on the motion, the links may loose some of the six degrees of freedom.





Planar mechanisms: When all the links of a mechanism have plane motion, it is called as a planar mechanism. All the links in a planar mechanism move in planes parallel to the reference plane.

Serial Mechanisms (Manipulators): Early manipulators were work holding devices in manufacturing operations so that the work piece could be manipulated or brought to different orientations with respect to the tool head. Welding robots of the auto industry and assembly robots of IC manufacture are examples.

Application of kutzbach criterion to Plane mechanisms

$$F = 3(n-1) - 2l - h$$

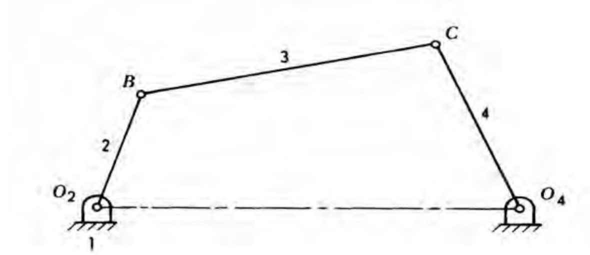
Where n=number of links; l= number of lower joints (or) pairs and h= number of higher pairs (or) joints

This is called the Kutzbach criterion for the mobility of a planar mechanism.

Inversion of Mechanism

A mechanism is one in which one of the links of a kinematic chain is fixed. Different mechanisms can be obtained by fixing different links of the same kinematic chain. These are called as inversions of the mechanism.

Inversions of Four Bar Chain



One of the most useful and most common mechanisms is the four-bar linkage. In this mechanism, the link which can make complete rotation is known as crank (link 2). The link which oscillates is known as rocker or lever (link 4). And the link connecting these two is known as coupler (link 3). Link 1 is the frame.

Inversions:

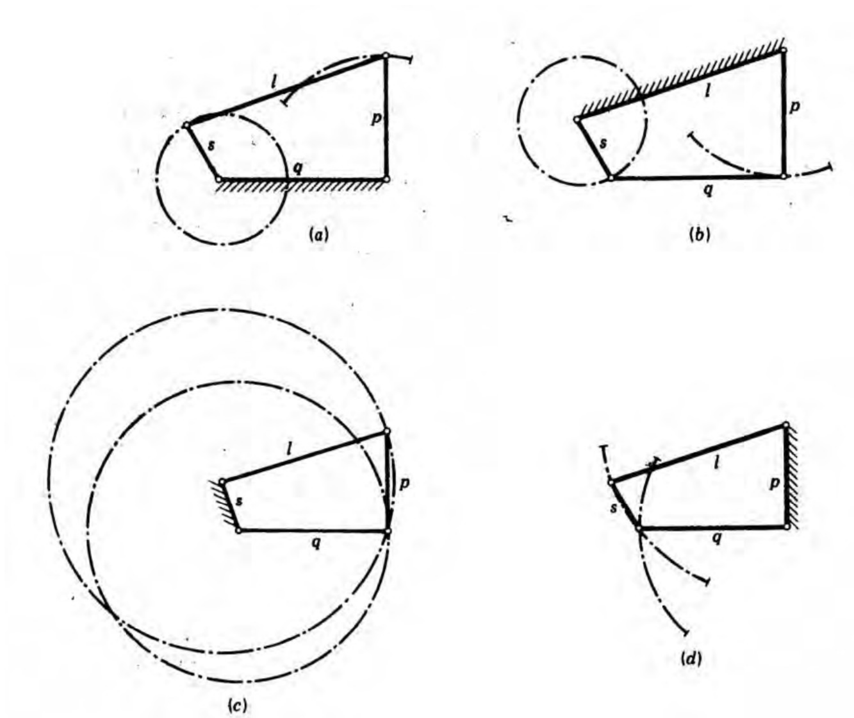


Fig.1.23 Inversions of four bar chain.

Crank-rocker mechanism: In this mechanism, either link 1 or link 3 is fixed. Link 2 (crank) rotates completely and link 4 (rocker) oscillates. It is similar to (a) or (b) of fig.1.23.

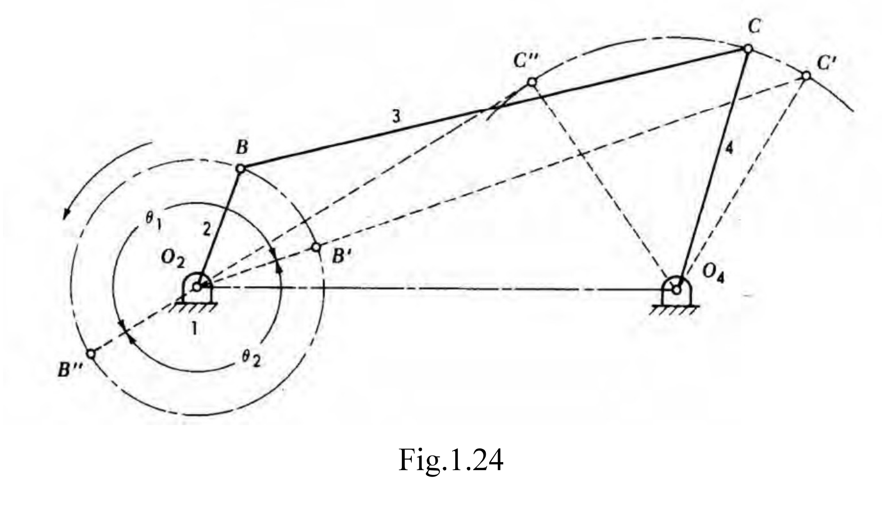


Fig.1.24

Double crank mechanism (Coupling rod of locomotive). This is one type of drag link mechanism, where, links 1& 3 are equal and parallel and links 2 & 4 are equal and parallel.

The mechanism of a coupling rod of a locomotive (also known as double crank mechanism) which consists of four links in the fig. in this mechanism, the links *AD* and *BC* (having equal length) act as cranks and are connected to the respective wheels. The link *CD* acts as a coupling rod and the link *AB* is fixed in order to maintain a constant centre to centre distance between them. This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.

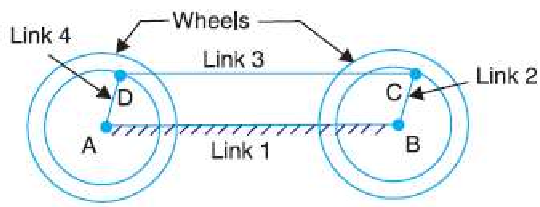


Fig.1.25

Double rocker mechanism. In this mechanism, link 4 is fixed. Link 2 makes complete rotation, whereas links 3 & 4 oscillate (Fig.1.23d)

Coupler Curves: The link connecting the driving crank with the follower crank in a four bar linkage is called the coupler. Similarly, in the case of a single slider crank mechanism the connecting rod is the coupler. During the motion of the mechanism any point attached to the coupler generates some path with respect to the fixed link. This path is called the coupler curve. The point, which generates the path is variously called the coupler point, trace point, tracing point, or tracer point.

An example of the coupler curves generated by different coupler points is given in figure below. Mechanisms can be designed to generate any curve.

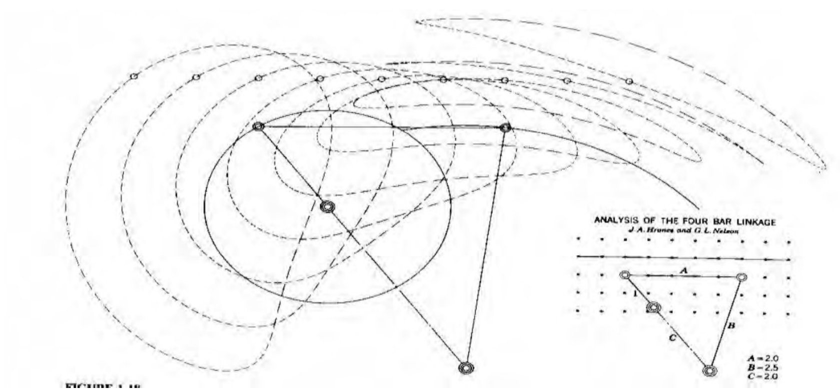


Fig. 1.26

Inversions of Single Slider Chain

Slider crank chain: This is a kinematic chain having four links. It has one sliding pair and three turning pairs. Link 2 has rotary motion and is called crank. Link 3 has got combined rotary and reciprocating motion and is called connecting rod. Link 4 has reciprocating motion and is called slider. Link 1 is frame (fixed). This mechanism is used to convert rotary motion to reciprocating and vice versa.

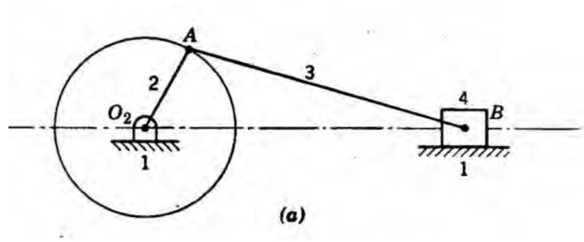
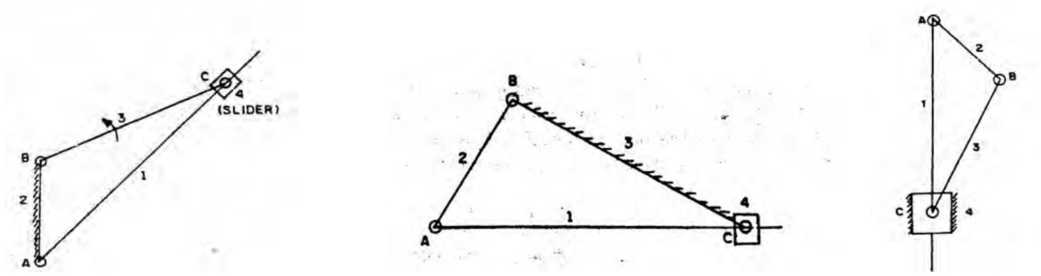


Fig1.27

Inversions of slider crank chain

Inversions of slider crank mechanism is obtained by fixing links 2, 3 and 4.



(a) crank fixed (b) connecting rod fixed (c) slider fixed

Fig.1.28

Quick return motion mechanisms.

Quick return mechanisms are used in machine tools such as shapers and power driven saws for the purpose of giving the reciprocating cutting tool a slow cutting stroke and a quick return stroke with a constant angular velocity of the driving crank.

Whitworth quick return motion mechanism–Inversion of slider crank mechanism.
This mechanism is mostly used in shaping and slotting machines. In this mechanism, the link *CD* (link 2) forming the turning pair is fixed, as shown in Fig. The link 2 corresponds to a crank in a reciprocating steam engine. The driving crank *CA* (link 3) rotates at a uniform angular speed. The slider (link 4) attached to the crank pin at *A* slides along the slotted bar *PA* (link 1) which oscillates at a pivoted point *D*. The connecting rod *PR* carries the ram at *R* to which a cutting tool is fixed. The motion of the tool is constrained along the line *RD* produced, *i.e.* along a line passing through *D* and perpendicular to *CD*.

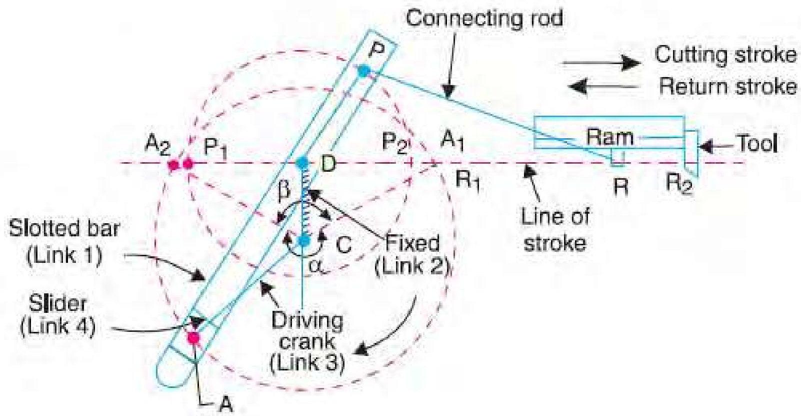


Fig.1.29

When the driving crank *CA* moves from the position *CA*1 to *CA*2 (or the link *DP* from the position *DP*1 to *DP*2) through an angle α in the clockwise direction, the tool moves from the left hand end of its stroke to the right hand end through a distance 2 *PD*.

Now when the driving crank moves from the position *CA*2 to *CA*1 (or the link *DP* from *DP*2 to *DP*1) through an angle β in the clockwise direction, the tool moves back from right hand end of its stroke to the left hand end.

A little consideration will show that the time taken during the left to right movement of the ram (*i.e*. during forward or cutting stroke) will be equal to the time taken by the driving crank to move from *CA*1 to *CA*2. Similarly, the time taken during the right to left movement of the ram (or during the idle or return stroke) will be equal to the time taken by the driving crank to move from *CA*2 to *CA*1.

Since the crank link *CA* rotates at uniform angular velocity therefore time taken during the cutting stroke (or forward stroke) is more than the time taken during the return stroke. In other words, the mean speed of the ram during cutting stroke is less than the mean speed during the return stroke.

The ratio between the time taken during the cutting and return strokes is given by

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360^\circ - \alpha}$$

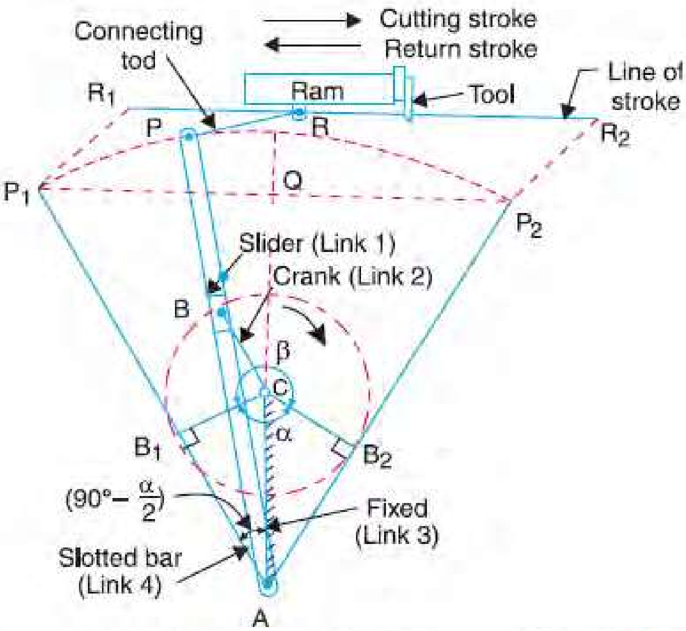
Crank and slotted lever quick return motion mechanism – Inversion of slider crank mechanism (connecting rod fixed).

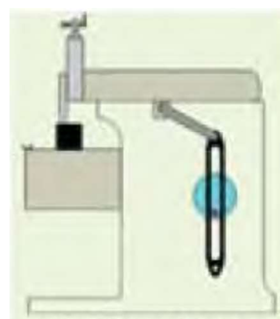
This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines. In this mechanism, the link *AC* (*i.e*. link 3) forming the turning pair is fixed, as shown in Fig. The link 3 corresponds to the connecting rod of a reciprocating steam engine. The driving crank *CB* revolves with uniform angular speed about the fixed centre *C*. A sliding block attached to the crank pin at *B* slides along the slotted bar *AP* and thus causes *AP* to oscillate about the pivoted point *A*. A short link *PR* transmits the motion from *AP* to the ram which carries the tool and reciprocates along the line of stroke *R*1*R*2. The line of stroke of the ram (*i.e.* *R*1*R*2) is perpendicular to *AC* produced.

We see that the angle β made by the forward or cutting stroke is greater than the angle α described by the return stroke. Since the crank rotates with uniform angular speed, therefore the return stroke is completed within shorter time. Thus it is called quick return motion mechanism.

In the extreme positions, *AP*1 and *AP*2 are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position *CB*1 to *CB*2 (or through an angle β) in the clockwise direction. The return stroke occurs when the crank rotates from the position *CB*2 to *CB*1 (or through angle α) in the clockwise direction. Since the crank has uniform angular speed, therefore,

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360° - \beta}$$





The Shaping Machine

Since the tool travels a distance of $R_1 R_2$ during cutting and return stroke, therefore travel of the tool or length of stroke

$$= R_1R_2 = P_1P_2 = 2P_1Q = 2AP_1 \sin \angle P_1AQ$$

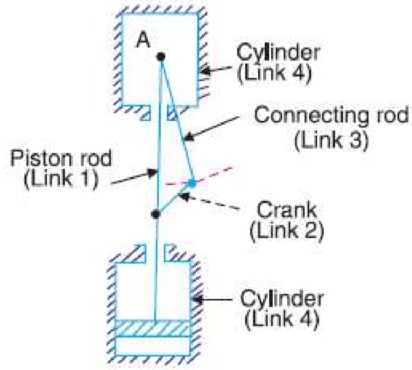
$$= 2AP_1 \sin\left(90° - \frac{\alpha}{2}\right) = 2AP \cos\frac{\alpha}{2}$$

$$= 2AP \times \frac{CB_1}{AC}$$

$$= 2AP \times \frac{CB}{AC}$$

Pendulum pump or bull engine–Inversion of slider crank mechanism (slider fixed).

In this mechanism, the inversion is obtained by fixing the cylinder or link 4 (*i.e.* sliding pair), as shown in Fig. In this case, when the crank (link 2) rotates, the connecting rod (link 3) oscillates about a pin pivoted to the fixed link 4 at *A* and the piston attached to the piston rod (link 1) reciprocates. The duplex pump which is used to supply feed water to boilers have two pistons attached to link 1, as shown in Fig.



VELOCITY AND ACCELERATION ANALYSIS OF MECHANISMS

In this, we shall discuss the relative velocity method for determining the velocity of different points in the mechanism. The study of velocity analysis is very important for determining the acceleration of points in the mechanisms.

Kinematics deals with study of relative motion between the various parts of the machines. Kinematics does not involve study of forces. Thus motion leads study of displacement, velocity and acceleration of a part of the machine. As dynamic forces are a function of acceleration and acceleration is a function of velocities, study of velocity and acceleration will be useful in the design of mechanism of a machine. The mechanism will be represented by a line diagram which is known as configuration diagram. The analysis can be carried out both by graphical method as well as analytical method.

Displacement: All particles of a body move in parallel planes and travel by same distance is known, linear displacement and is denoted by 'x'. A body rotating about a fired point in such a way that all particular move in circular path angular displacement and is denoted by 'θ'.

Velocity: Rate of change of displacement is velocity. Velocity can be linear velocity of angular velocity.

Linear velocity is Rate of change of linear displacement= $V = \frac{dx}{dt}$

Angular velocity is Rate of change of angular displacement = $\omega = \frac{d\theta}{dt}$

Relation between linear velocity and angular velocity.

$x = r\theta$ **$V = r\omega$**

$$\frac{dx}{dt} = r\frac{d\theta}{dt} \qquad \boldsymbol{\omega} = \frac{d\theta}{dt}$$

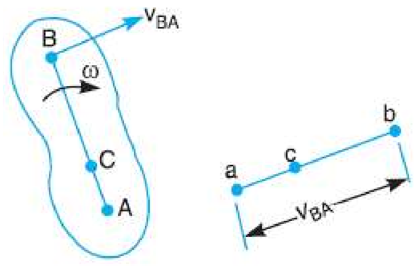
Acceleration: Rate of change of velocity

$$f = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$ Linear Acceleration (Rate of change of linear velocity)

Thirdly $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ Angular Acceleration (Rate of change of angular velocity)

Motion of a link

Consider two points A and B on a rigid link AB, as shown in Fig. (a). Let one of the extremities (B) of the link move relative to A, in a clockwise direction. Since the distance from A to B remains the same, therefore there can be no relative motion between A and B, along the line AB. It is thus obvious, that the relative motion of B with respect to A must be perpendicular to AB. Hence **velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points on the configuration (or space) diagram.**



(*a*) (*b*)

The relative velocity of *B* with respect to *A* (*i.e.* *v*BA) is represented by the vector *ab* and is perpendicular to the line *AB* as shown in Fig. (*b*).

Let ω = Angular velocity of the link *A B* about *A*.

We know that the velocity of the point *B* with respect to *A*.

$$v_{BA} = \overline{ab} = \omega . AB$$

Similarly, the velocity of any point *C* on *A B* with respect to *A*.

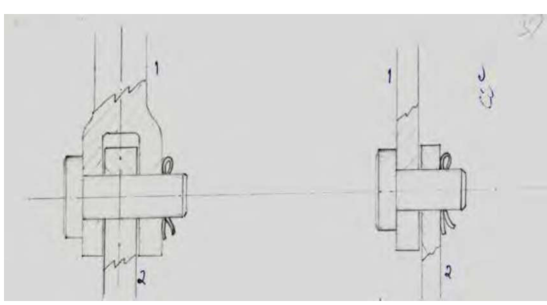
$$v_{CA} = \overline{ac} = \omega . AC$$

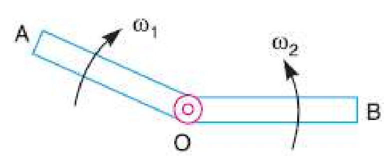
From the above two equations

$$\frac{v_{CA}}{v_{BA}} = \frac{\overline{ac}}{\overline{ab}} = \frac{\omega . AC}{\omega . AB} = \frac{AC}{AB}$$

Thus, we see from above equation that the point *c* on the vector *ab* divides it in the same ratio as *C* divides the link *AB*.

Rubbing Velocity at a Pin Joint





The links in a mechanism are mostly connected by means of pin joints. The rubbing velocity is defined as **the algebraic sum between the angular velocities of the two links which are connected by pin joints, multiplied by the radius of the pin.**

Consider two links *OA* and *OB* connected by a pin joint at *O* as shown in Fig.
Let
$\omega 1$ = Angular velocity of the link *OA* or the angular velocity of the point *A* with respect to O
$\omega 2$ = Angular velocity of the link *OB* or the angular velocity of the point *B* with respect to *O*, and
r = Radius of the pin.

According to the definition,

Rubbing velocity at the pin joint *O* is given by the formula
$=(\omega 1-\omega 2)*r$, if the links move in the same direction
$= (\omega 1+\omega 2)*r$, if the links move in the opposite direction
where ω_1 = angular velocity of link 1
ω_2 = angular velocity of link 2
r = radius of the pin

Note : When the pin connects one sliding member and the other turning member, the angular velocity of the sliding member is zero. In such cases,

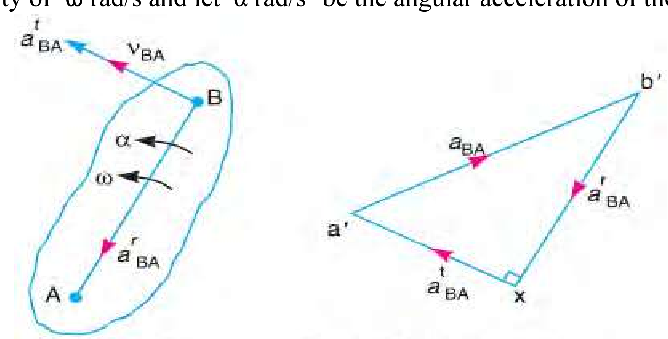
Rubbing velocity at the pin joint = ωr
where ω=Angular velocity of the turning member, and
r = Radius of the pin.

<u>Acceleration in mechanisms</u> (Introduction)

We have discussed in the previous chapter the velocities of various points in the mechanisms. Now we shall discuss the acceleration of points in the mechanisms. The acceleration analysis plays a very important role in the development of machines and mechanisms.

Acceleration diagram for a link

Consider two points *A* and *B* on a rigid link as shown in Fig. (*a*). Let the point *B* moves with respect to *A*, with an angular velocity of ω rad/s and let α rad/s^2 be the angular acceleration of the link *AB*.



(*a*) Link. (*b*) Acceleration diagram.

We have already discussed that acceleration of a particle whose velocity changes both in magnitude and direction at any instant has the following two components:
1. The *centripetal or radial component,* which is perpendicular to the velocity of the particle at the given instant.
2. The *tangential component,* which is parallel to the velocity of the particle at the given instant.
Thus for a link *AB*, the velocity of point *B* with respect to *A* (*i.e.* vBA) is perpendicular to the link *AB* as shown in Fig.(*a*). Since the point *B* moves with respect to *A* with an angular velocity of ω rad/s, therefore centripetal or radial component of the acceleration of *B* with respect to *A*,

$$a^r_{BA} = \omega^2 \times \text{Length of link } AB = \omega^2 \times AB = v^2_{BA} / AB \qquad \ldots\left(\because \omega = \frac{v_{BA}}{AB}\right)$$

This radial component of acceleration acts perpendicular to the velocity v_{BA}, In other words, it acts *parallel* to the link *AB.*

We know that tangential component of the acceleration of *B* with respect to *A*,

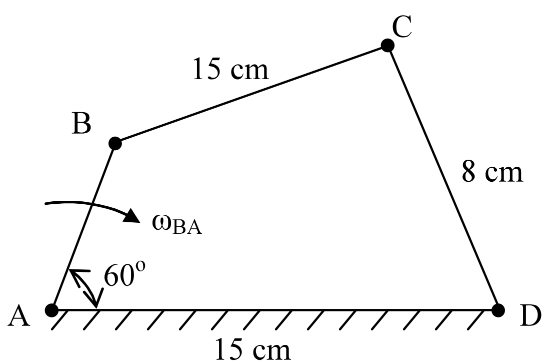
$$a^t_{BA} = \alpha \times \text{Length of the link } AB = \alpha \times AB$$

This tangential component of acceleration acts parallel to the velocity vBA. In other words, it acts *perpendicular* to the link *AB*.

In order to draw the acceleration diagram for a link *AB*, as shown in Fig. (*b*), from any point *b'*, draw vector *b'x parallel to BA* to represent the radial component of acceleration of *B* with respect to *A i.e.* a^r_{BA} from point *x* draw vector *xa'* perpendicular to *B A* to represent the tangential component of acceleration of *B* with respect to *A i.e.* a^t_{BA}. *Join b' a'.* The vector *b' a'* (known as *acceleration image* of the link *AB*) represents the total acceleration of *B* with respect to *A* (*i.e.* aBA) and it is the vector sum of radial component (a^r_{BA}) and tangential component (a^t_{BA}) of acceleration.

Exercise Problems:

1. In a four bar chain ABCD link AD is fixed and in 15 cm long. The crank AB is 4 cm long rotates at 180 rpm (cw) while link CD rotates about D is 8 cm long BC = AD and ∠BAD = 60^o. Find angular velocity of link CD.



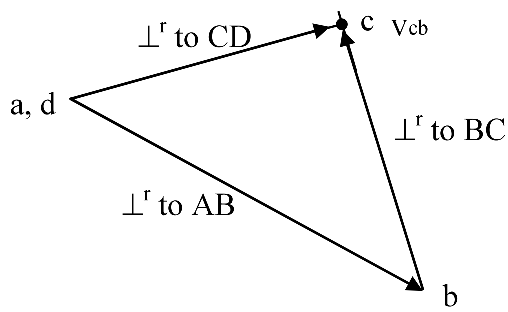
Configuration Diagram

Velocity vector diagram

$$V_b = \omega r = \omega_{ba} \text{ x } AB = \frac{2\pi x 120}{60} x4 = 50.24 \text{ cm/sec}$$

Choose a suitable scale

$$1 \text{ cm} = 20 \text{ m/s} = \overrightarrow{ab}$$



$V_{cb} = \overrightarrow{bc}$

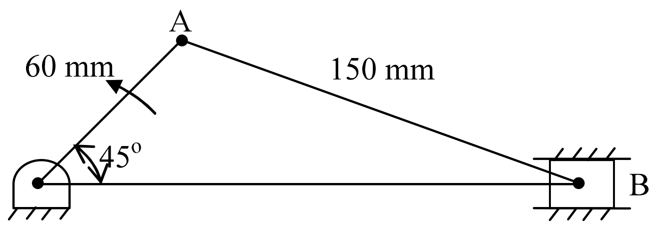
$V_c = \overrightarrow{dc} = 38$ cm/sec $= V_{cd}$

We know that $V = \omega R$

$V_{cd} = \omega_{CD} \times CD$

$\omega_{cD} = \frac{V_{cd}}{CD} = \frac{38}{8} = 4.75$ rad/sec (cw)

2. In a crank and slotted lever mechanism crank rotates of 300 rpm in a counter clockwise direction. Find
   (i) Angular velocity of connecting rod and
   (ii) Velocity of slider.



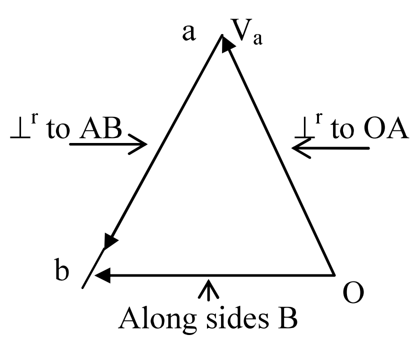
Configuration diagram

Step 1: Determine the magnitude and velocity of point A with respect to 0,

$$V_A = \omega_{O1A} \times O_2A = \frac{2\pi \times 300}{60} \times 60$$

$$= 600\ \pi \text{ mm/sec}$$

Step 2: Choose a suitable scale to draw velocity vector diagram.



Velocity vector diagram

$V_{ab} = \overrightarrow{ab} = 1300$mm/sec

$$\omega_{ba} = \frac{V_{ba}}{BA} = \frac{1300}{150} = 8.66 \text{ rad/sec}$$

$V_b = \overrightarrow{ob}$ velocity of slider

Note: Velocity of slider is along the line of sliding.

3. In a four bar mechanism, the dimensions of the links are as given below:

AB = 50 mm, BC = 66 mm

CD = 56 mm and AD = 100 mm

At a given instant when $\angle DAB = 60^\circ$ the angular velocity of link AB is 10.5 rad/sec in CCW direction.

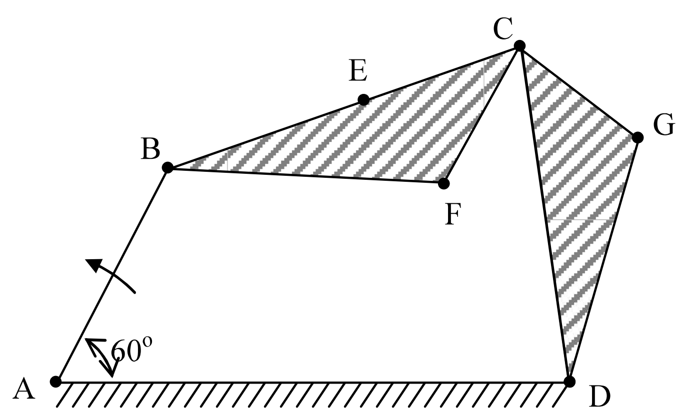
Determine,

i) Velocity of point C
ii) Velocity of point E on link BC when BE = 40 mm
iii) The angular velocity of link BC and CD
iv) The velocity of an offset point F on link BC, if BF = 45 mm, CF = 30 mm and BCF is read clockwise.
v) The velocity of an offset point G on link CD, if CG = 24 mm, DG = 44 mm and DCG is read clockwise.
vi) The velocity of rubbing of pins A, B, C and D. The ratio of the pins are 30 mm, 40 mm, 25 mm and 35 mm respectively.

Solution:

Step -1: Construct the configuration diagram selecting a suitable scale.

Scale: 1 cm = 20 mm



Step – 2: Given the angular velocity of link AB and its direction of rotation determine velocity of point with respect to A (A is fixed hence, it is zero velocity point).

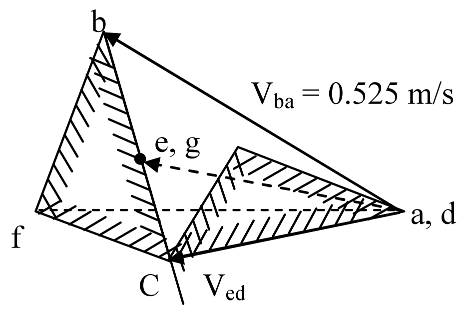
$V_{ba} = \omega_{BA} \times BA$

$= 10.5 \times 0.05 = 0.525$ m/s

Step – 3: To draw velocity vector diagram choose a suitable scale, say 1 cm = 0.2 m/s.

- First locate zero velocity points.

- Draw a line $\perp^r$ to link AB in the direction of rotation of link AB (CCW) equal to 0.525 m/s.



- From b draw a line $\perp^r$ to BC and from d. Draw d line $\perp^r$ to CD to interest at C.
- V_{cb} is given vector bc V_{bc} = 0.44 m/s
- V_{cd} is given vector dc V_{cd} = 0.39 m/s

Step – 4: To determine velocity of point E (Absolute velocity) on link BC, first locate the position of point E on velocity vector diagram. This can be done by taking corresponding ratios of lengths of links to vector distance i.e.

$$\frac{be}{bc} = \frac{BE}{BC}$$

$$\therefore \text{ be} = \frac{BE}{BC} \text{ x } V_{cb} = \frac{0.04}{0.066} \text{ x } 0.44 = 0.24 \text{ m/s}$$

Join e on velocity vector diagram to zero velocity points a, d / vector $\overrightarrow{de}$ = V_e = 0.415 m/s.

Step 5: To determine angular velocity of links BC and CD, we know V_{bc} and V_{cd}.

$$\therefore V_{bc} = \omega_{BC} \text{ x } BC$$

$$\therefore \omega_{BC} = \frac{V_{bc}}{BC} = \frac{0.44}{0.066} = 6.6 \; r/s \;.\; (cw)$$

Similarly, $V_{cd} = \omega_{CD}$ x CD

$$\therefore \omega_{CD} = \frac{V_{cd}}{CD} = \frac{0.39}{0.056} = 6.96 \text{ r/s (CCW)}$$

Step – 6: To determine velocity of an offset point F

- Draw a line $\perp^r$ to CF from C on velocity vector diagram.
- Draw a line $\perp^r$ to BF from b on velocity vector diagram to intersect the previously drawn line at ‘f’.
- From the point f to zero velocity point a, d and measure vector fa to get

  V_f = 0.495 m/s.

Step – 7: To determine velocity of an offset point.

- Draw a line $\perp^r$ to GC from C on velocity vector diagram.
- Draw a line $\perp^r$ to DG from d on velocity vector diagram to intersect previously drawn line at g.
- Measure vector dg to get velocity of point G.

$$V_g = \overrightarrow{dg} = 0.305 \text{ m/s}$$

Step – 8: To determine rubbing velocity at pins

- Rubbing velocity at pin A will be

  $V_{pa} = \omega_{ab}$ x r of pin A

  $V_{pa} = 10.5 \times 0.03 = 0.315$ m/s

- Rubbing velocity at pin B will be

  $V_{pb} = (\omega_{ab} + \omega_{cb}) \times r_{pb}$ of point at B.

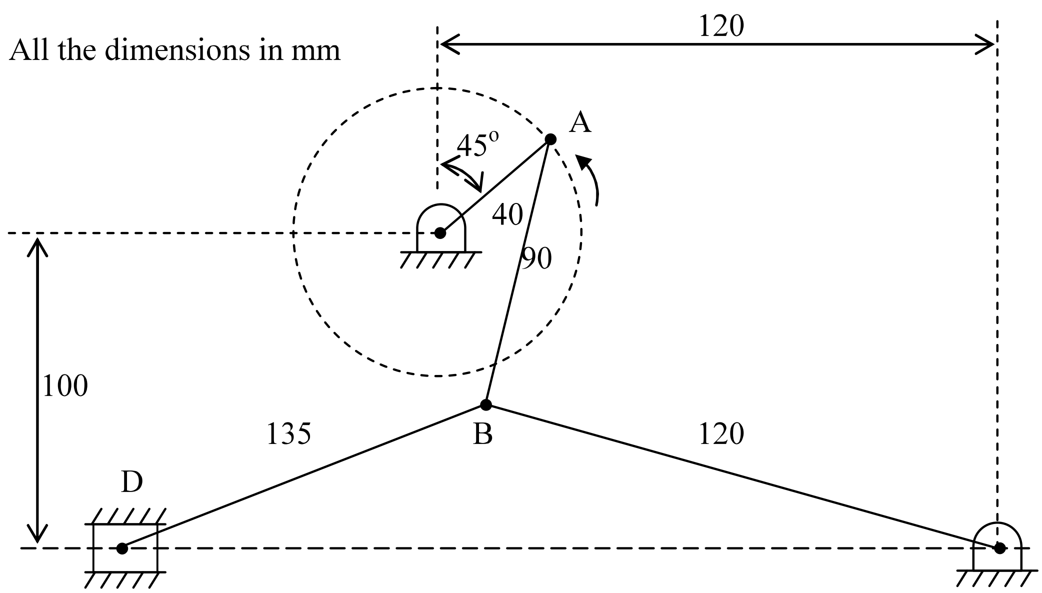
  [ω_{ab} CCW and ω_{cb}CW]

  $V_{pb} = (10.5 + 6.6) \times 0.04 = 0.684$ m/s.

- Rubbing velocity at point C will be = 6.96 x 0.035 = 0.244 m/s

4. Figure below shows a toggle mechanism in which the crank OA rotates at 120 rpm. Find the velocity and acceleration of the slider D.

Solution:



Configuration Diagram

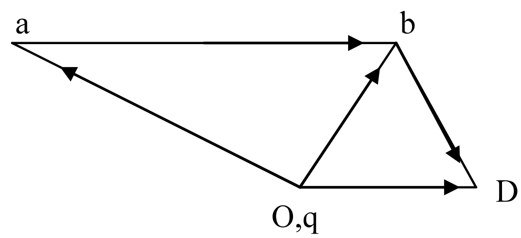
Step 1: Draw the configuration diagram choosing a suitable scale.
Step 2: Determine velocity of point A with respect to O.

$$V_{ao} = \omega_{OA} \times OA$$

$$V_{ao} = \frac{2\pi \times 120}{60} = 0.4 = 5.024\ m/s$$

Step 3: Draw the velocity vector diagram.

- Choose a suitable scale
- Mark zero velocity points O,q
- Draw vector $\vec{oa} \perp^r$ to link OA and magnitude = 5.024 m/s.



Velocity vector diagram

- From a draw a line $\perp^r$ to AB and from q draw a line $\perp^r$ to QB to intersect at b.

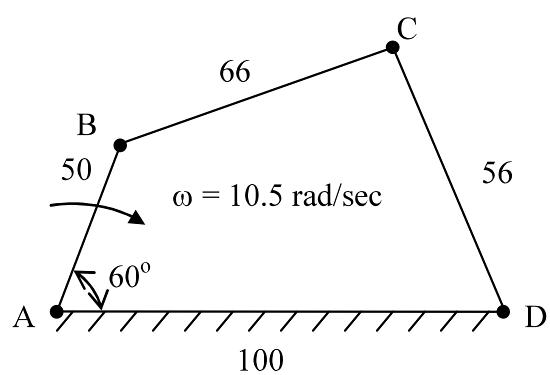
$$\vec{ab} = V_{ba} \text{ and } \vec{qb} = V_{bq}.$$

- Draw a line $\perp^r$ to BD from b from q draw a line along the slide to intersect at d.

$$\vec{dq} = V_d \text{ (slider velocity)}$$

5. For a 4-bar mechanism shown in figure draw velocity and acceleration diagram.

All dimensions are in mm



Solution:

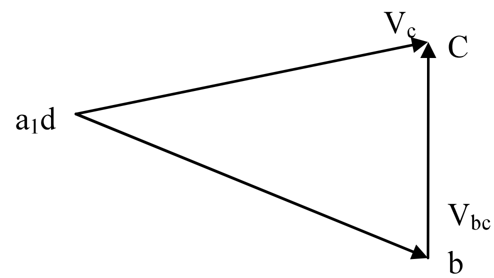
Step 1: Draw configuration diagram to a scale.

Step 2: Draw velocity vector diagram to a scale.

$$V_b = \omega_2 \times AB$$

$$V_b = 10.5 \times 0.05$$

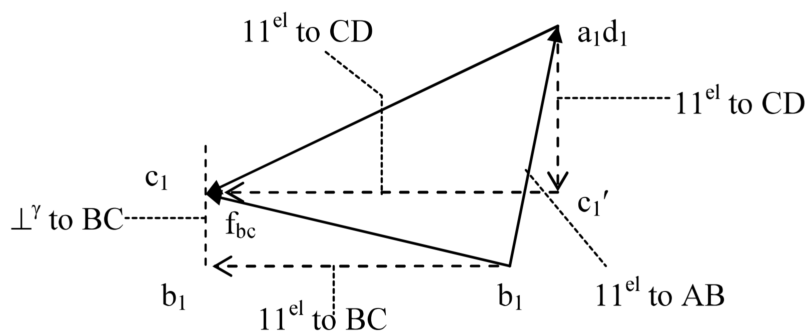
$$V_b = 0.525\ m/s$$



Step 3: Prepare a table as shown below:

Sl. No.	Link	Magnitude	Direction	Sense
1.	AB	$f^c = \omega^2_{AB}r$ $f^c = (10.5)^2/0.525$ $f^c = 5.51$ m/s^2	Parallel to AB	→ A
2.	BC	$f^c = \omega^2_{BC}r$ $f^c = 1.75$ $f^t = \alpha r$	Parallel to BC $\perp^r$ to BC	→ B –
3.	CD	$f^c = \omega^2_{CD}r$ $f^c = 2.75$ $f^t = ?$	Parallel to DC $\perp^r$ to DC	→ D –

Step 4: Draw the acceleration diagram.



- Choose a suitable scale to draw acceleration diagram.
- Mark the zero acceleration point a_1d_1.
- Link AB has only centripetal acceleration. Therefore, draw a line parallel to AB and toward A from a_1d_1 equal to 5.51 m/s^2 i.e. point b_1.
- From b_1 draw a vector parallel to BC points towards B equal to 1.75 m/s^2 ($b^1{}_1$).

- From b^1_1 draw a line $\perp^r$ to BC. The magnitude is not known.
- From a_1d_1 draw a vector parallel to AD and pointing towards D equal to 2.72 m/s^2 i.e. point c_1.
- From c^1_1 draw a line $\perp^r$ to CD to intersect the line drawn $\perp^r$ to BC at c_1, $\overrightarrow{d_1c_1}$ = f_{CD} and $\overrightarrow{b_1c_1} = f_{bc}$.

To determine angular acceleration.

$$\alpha_{BC} = \frac{f^t_{bc}}{BC} = \frac{\overrightarrow{c_1b^1_1}}{BC} = 34.09\,\text{rad/sec (CCW)}$$

$$\alpha_{CD} = \frac{f^t_{cd}}{CD} = \frac{\overrightarrow{c_1c^1_1}}{CD} = 79.11\,\text{rad/sec(CCW)}$$

BELT DRIVES

Introduction

The belts or ropes are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds.

The amount of power transmitted depends upon the following factors:

1. The velocity of the belt.
2. The tension under which the belt is placed on the pulleys.
3. The conditions under which the belt is used.

Selection of a Belt Drive

Following are the various important factors upon which the selection of a belt drive depends:

1. Speed of the driving and driven shafts,
2. Speed reduction ratio,
3. Power to be transmitted,
4. Centre distance between the shafts,
5. Positive drive requirements,
6. Shafts layout,
7. Space available, and
8. Service conditions.

Types of Belt Drives

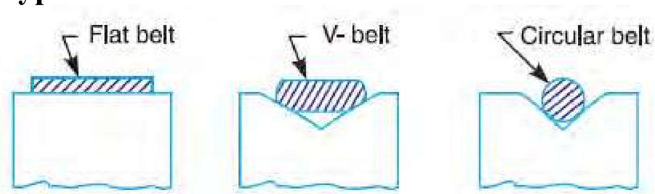
The belt drives are usually classified into the following three groups :

1. ***Light drives***. These are used to transmit small powers at belt speeds upto about 10 m/s, as in agricultural machines and small machine tools.

2. ***Medium drives***. These are used to transmit medium power at belt speeds over 10 m/s but up to 22 m/s, as in machine tools.

3. ***Heavy drives***. These are used to transmit large powers at belt speeds above 22 m/s, as in compressors and generators.

Types of Belts



(*a*) Flat belt. (*b*) V-belt. (*c*) Circular belt.

Though there are many types of belts used these days, yet the following are important from the subject point of view:

1. ***Flat belt.*** The flat belt, as shown in Fig. (*a*), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another when the two pulleys are not more than 8 metres apart.

2. ***V-belt.*** The V-belt, as shown in Fig. (*b*), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another, when the two pulleys are very near to each other.

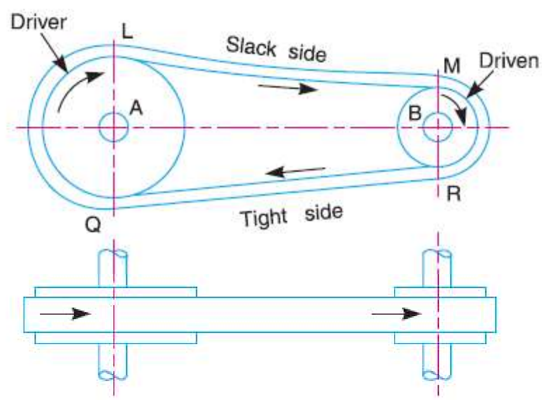
3. *Circular belt or rope*. The circular belt or rope, as shown in Fig. (*c*), is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are more than 8 meters apart.

If a huge amount of power is to be transmitted, then a single belt may not be sufficient. In such a case, wide pulleys (for V-belts or circular belts) with a number of grooves are used. Then a belt in each groove is provided to transmit the required amount of power from one pulley to another.

Types of Flat Belt Drives

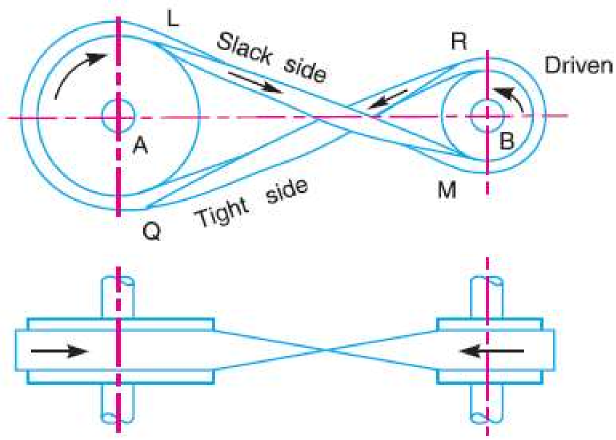
The power from one pulley to another may be transmitted by any of the following types of belt drives:

1. *Open belt drive*. The open belt drive, as shown in Fig. 11.3, is used with shafts arranged parallel and rotating in the same direction. In this case, the driver *A* pulls the belt from one side (*i.e.* lower side *RQ*) and delivers it to the other side (*i.e.* upper side *LM*). Thus the tension in the lower side belt will be more than that in the upper side belt. The lower side belt (because of more tension) is known as ***tight side*** whereas the upper side belt (because of less tension) is known as ***slack side***, as shown in Fig.

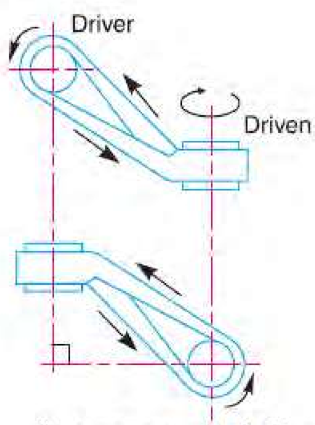


2. *Crossed or twist belt drive*. The crossed or twist belt drive, as shown in Fig. 11.4, is used with shafts arranged parallel and rotating in the opposite directions. In this case, the driver pulls the belt from one side (*i.e.* *RQ*) and delivers it to the other side (*i.e.* *LM*). Thus the tension in the belt *RQ* will be more than that in the belt *LM*. The belt *RQ* (because of more tension) is known as ***tight side,*** whereas the belt *LM* (because of less tension) is known as ***slack side***, as shown in Fig.

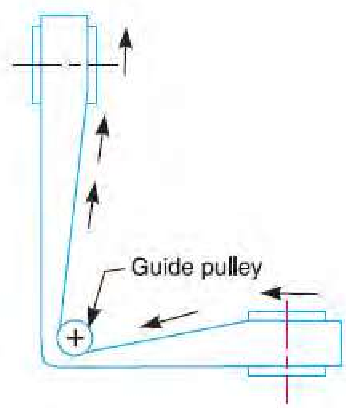
A little consideration will show that at a point where the belt crosses, it rubs against each other and there will be excessive wear and tear. In order to avoid this, the shafts should be placed at a maximum distance of 20 *b*, where *b* is the width of belt and the speed of the belt should be less than 15 m/s.



3. ***Quarter turn belt drive*.** The quarter turn belt drive also known as right angle belt drive, as shown in Fig. (*a*), is used with shafts arranged at right angles and rotating in one definite direction. In order to prevent the belt from leaving the pulley, the width of the face of the pulley should be greater or equal to 1.4 *b*, where *b* is the width of belt. In case the pulleys cannot be arranged, as shown in Fig. (*a*), or when the reversible is desired, then a ***quarter turn belt drive with guide pulley***, as shown in Fig. (*b*), may be used.

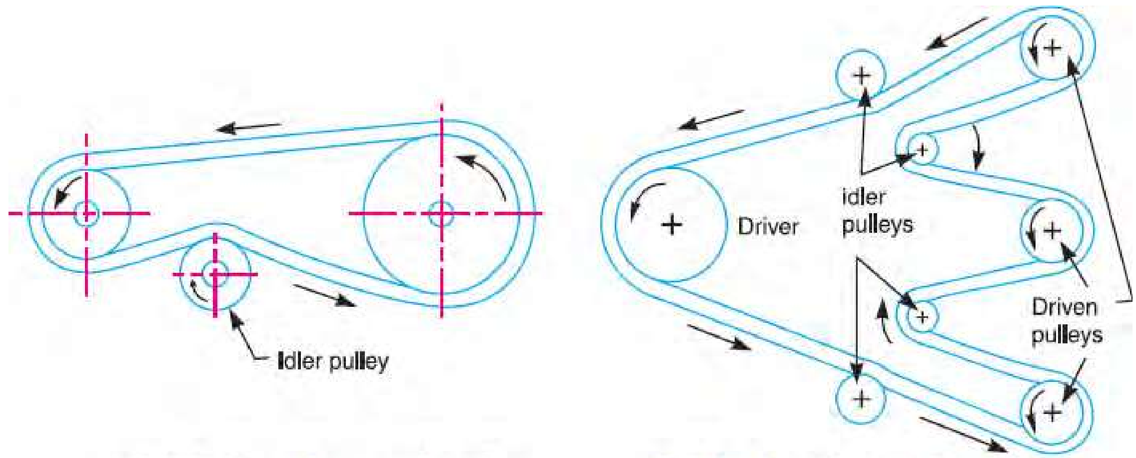


(*a*) Quarter turn belt drive.



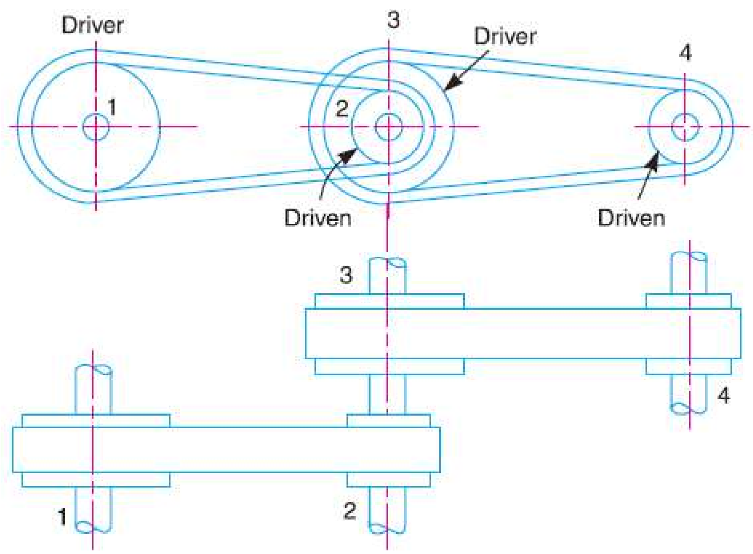
(*b*) Quarter turn belt drive with guide pulley.

4. ***Belt drive with idler pulleys***. A belt drive with an idler pulley, as shown in Fig. (*a*), is used with shafts arranged parallel and when an open belt drive cannot be used due to small angle of contact on the smaller pulley. This type of drive is provided to obtain high velocity ratio and when the required belt tension cannot be obtained by other means.
When it is desired to transmit motion from one shaft to several shafts, all arranged in parallel, a belt drive with many idler pulleys, as shown in Fig. (*b*), may be employed.

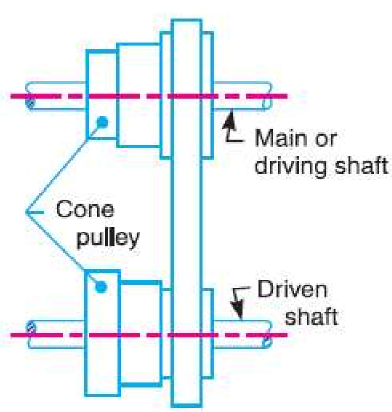


(*a*) Belt drive with single idler pulley. (*b*) Belt drive with many idler pulleys.

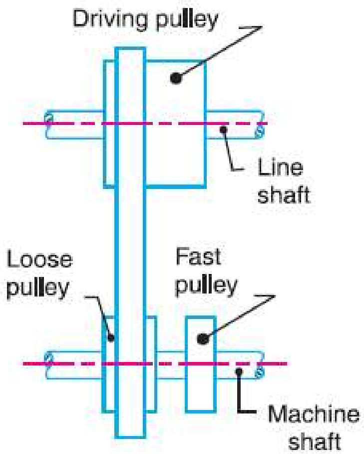
5. ***Compound belt drive***. A compound belt drive, as shown in Fig., is used when power is transmitted from one shaft to another through a number of pulleys.



6. ***Stepped or cone pulley drive***. A stepped or cone pulley drive, as shown in Fig, is used for changing the speed of the driven shaft while the main or driving shaft runs at constant speed. This is accomplished by shifting the belt from one part of the steps to the other.



7. ***Fast and loose pulley drive***. A fast and loose pulley drive, as shown in Fig., is used when the driven or machine shaft is to be started or stopped when ever desired without interfering with the driving shaft. A pulley which is keyed to the machine shaft is called ***fast pulley*** and runs at the same speed as that of machine shaft. A loose pulley runs freely over the machine shaft and is incapable of transmitting any power. When the driven shaft is required to be stopped, the belt is pushed on to the loose pulley by means of sliding bar having belt forks.



Velocity Ratio of Belt Drive

It is the ratio between the velocities of the driver and the follower or driven. It may be expressed, mathematically, as discussed below:

Let $d1$ = Diameter of the driver,

$d2$ = Diameter of the follower,

$N1$ = Speed of the driver in r.p.m., and

$N2$ = Speed of the follower in r.p.m.

Length of the belt that passes over the driver, in one minute = $\pi\, d1N1$

Similarly, length of the belt that passes over the follower, in one minute = $\pi\, d2\, N2$

Since the length of belt that passes over the driver in one minute is equal to the length of belt that passes over the follower in one minute, therefore

$\pi\, d1\, N1 = \pi\, d2\, N2$

Velocity ratio, $\frac{N_2}{N_1} = \frac{d_1}{d_2}$

When the thickness of the belt (t) is considered, then velocity ratio,

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$$

The velocity ratio of a belt drive may also be obtained as discussed below :

We know that peripheral velocity of the belt on the driving pulley,

$$v_1 = \frac{\pi d_1 . N_1}{60} \text{ m/s}$$

and peripheral velocity of the belt on the driven or follower pulley,

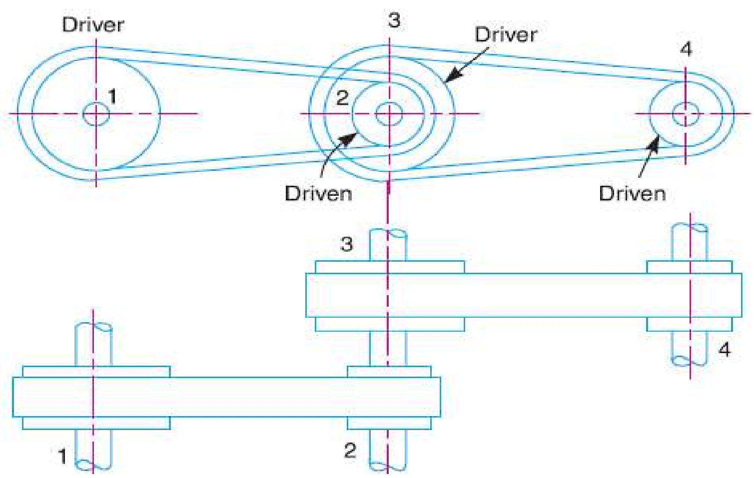
$$v_2 = \frac{\pi d_2 . N_2}{60} \text{ m/s}$$

When there is no slip, then $v_1 = v_2$.

$$\therefore \quad \frac{\pi d_1 . N_1}{60} = \frac{\pi d_2 . N_2}{60} \quad \text{or} \quad \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

Velocity Ratio of a Compound Belt Drive

Sometimes the power is transmitted from one shaft to another, through a number of pulleys, as shown in fig. Consider a pulley 1 driving the pulley 2. Since the pulleys 2 and 3 are keyed to the same shaft, therefore the pulley 1 also drives the pulley 3 which, in turn, drives the pulley 4.



Let

$d1$ = Diameter of the pulley 1,

$N1$ = Speed of the pulley 1 in r.p.m.,

$d2$, $d3$, $d4$, and $N2$, $N3$, $N4$= Corresponding values for pulleys 2, 3 and 4.

We know that velocity ratio of pulleys 1 and 2,

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}$$

Similarly, velocity ratio of pulleys 3 and 4,

$$\frac{N_4}{N_3} = \frac{d_3}{d_4}$$

Multiplying the above equations gives

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} = \frac{d_1}{d_2} \times \frac{d_3}{d_4}$$

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \quad ...(\because N_2 = N_3, \text{ being keyed to the same shaft})$$

Slip of Belt

In the previous articles, we have discussed the motion of belts and shafts assuming a firm frictional grip between the
belts and the shafts. But sometimes, the frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This may also cause some forward motion of the belt without carrying the driven pulley with it. This is called ***slip of the belt*** and is generally expressed as a percentage.

The result of the belt slipping is to reduce the velocity ratio of the system. As the slipping of the belt is a common phenomenon, thus the belt should never be used where a definite velocity ratio is of importance.

Let $s_1\%$ = Slip between the driver and the belt, and

$s_2\%$ = Slip between the belt and the follower.

∴ Velocity of the belt passing over the driver per second

$$v = \frac{\pi d_1 . N_1}{60} - \frac{\pi d_1 . N_1}{60} \times \frac{s_1}{100} = \frac{\pi d_1 . N_1}{60}\left(1 - \frac{s_1}{100}\right)$$

and velocity of the belt passing over the follower per second,

$$\frac{\pi d_2 . N_2}{60} = v - v \times \frac{s_2}{100} = v\left(1 - \frac{s_2}{100}\right)$$

Substituting the value of v from equation (i),

$$\frac{\pi d_2 N_2}{60} = \frac{\pi d_1 N_1}{60}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{s_1}{100} - \frac{s_2}{100}\right) \quad \ldots \left(\text{Neglecting } \frac{s_1 \times s_2}{100 \times 100}\right)$$

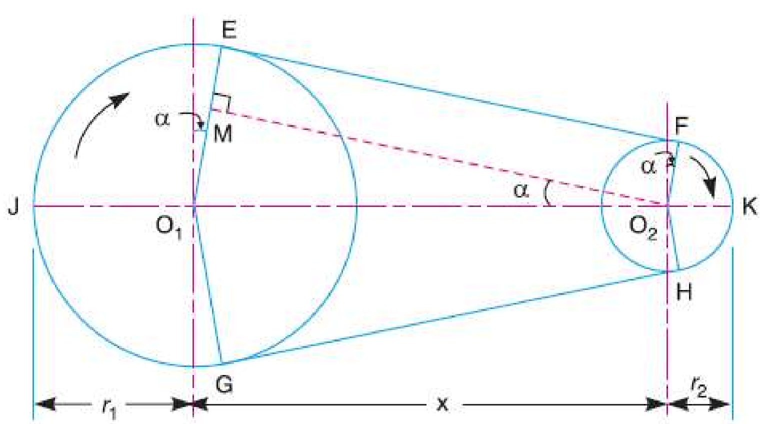
$$= \frac{d_1}{d_2}\left(1 - \frac{s_1 + s_2}{100}\right) = \frac{d_1}{d_2}\left(1 - \frac{s}{100}\right)$$

... (where $s = s_1 + s_2$, *i.e.* total percentage of slip)

If thickness of the belt (t) is considered, then

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{s}{100}\right)$$

Length of an open Belt Drive



We have already discussed that in an open belt drive, both the pulleys rotate in the ***same*** direction as shown in Fig.

Let
r_1 and r_2 = Radii of the larger and smaller pulleys,
x = Distance between the centres of two pulleys (*i.e.* $O_1 O_2$), and
L = Total length of the belt.

Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H as shown in Fig. Through O_2, draw $O_2 M$ parallel to FE.

From the geometry of the figure, we find that $O_2 M$ will be perpendicular to $O_1 E$.
Let the angle $MO_2 O_1 = \alpha$ radians.

We know that the length of the belt,
L = Arc GJE + EF + Arc FKH + HG

= 2 (Arc JE + EF + Arc FK)

From the geometry of the figure, we find that

$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E - EM}{O_1 O_2} = \frac{r_1 - r_2}{x}$$

Since α is very small, therefore putting

$$\sin\alpha = \alpha \text{ (in radians)} = \frac{r_1 - r_2}{x}$$

$$\therefore \quad \text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right)$$

Similarly $$\text{Arc } FK = r_2\left(\frac{\pi}{2} - \alpha\right)$$

and $$EF = MO_2 = \sqrt{(O_1 O_2)^2 - (O_1 M)^2} = \sqrt{x^2 - (r_1 - r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem,

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2 + ...\right] = x - \frac{(r_1 - r_2)^2}{2x}$$

$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 - r_2)^2}{2x} + r_2\left(\frac{\pi}{2} - \alpha\right)\right]$$

$$= 2\left[r_1 \times \frac{\pi}{2} + r_1.\alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2 \times \frac{\pi}{2} - r_2.\alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

Substituting the value of $\alpha = \dfrac{r_1 - r_2}{x}$

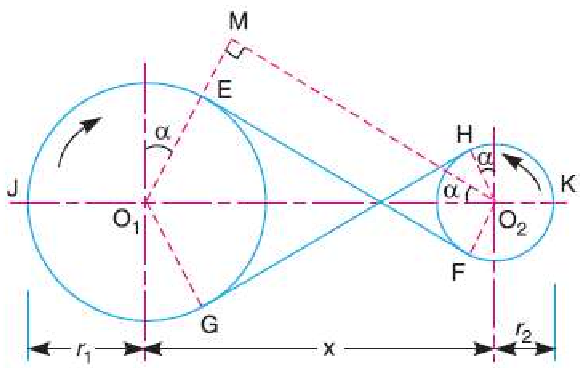
$$L = \pi(r_1 + r_2) + 2 \times \frac{(r_1 - r_2)}{x} \times (r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{2(r_1 - r_2)^2}{x} + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x} \quad \text{...(In terms of pulley radii)}$$

$$= \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x} \quad \text{...(In terms of pulley diameters)}$$

Length of a Cross Belt Drive



We have already discussed that in a cross belt drive, both the pulleys rotate in ***opposite*** directions as shown in Fig.
Let $r1$ and $r2$ = Radii of the larger and smaller pulleys,
x = Distance between the centres of two pulleys (*i.e.* $O1\ O2$), and
L = Total length of the belt.
Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H, as shown in Fig. Through $O2$, draw $O2M$ parallel to FE.
From the geometry of the figure, we find that $O2M$ will be perpendicular to $O1E$.
Let the angle $MO2\ O1$ = α radians
We know that the length of the belt,
L = Arc GJE + EF + Arc FKH + HG
= 2 (Arc JE + EF + Arc FK)

From the geometry of the figure, we find that

$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E + EM}{O_1 O_2} = \frac{r_1 + r_2}{x}$$

Since α is very small, therefore putting

$$\sin\alpha = \alpha \text{ (in radians)} = \frac{r_1 + r_2}{x}$$

$$\therefore \quad \text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right)$$

Similarly $\quad$ Arc $FK = r_2\left(\frac{\pi}{2} + \alpha\right)$

$$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1 M)^2} = \sqrt{x^2 - (r_1 + r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 + r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem,

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 + r_2}{x}\right)^2 + \ldots\right] = x - \frac{(r_1 + r_2)^2}{2x}$$

$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 + r_2)^2}{2x} + r_2\left(\frac{\pi}{2} + \alpha\right)\right]$$

$$= 2\left[r_1 \times \frac{\pi}{2} + r_1 . \alpha + x - \frac{(r_1 + r_2)^2}{2x} + r_2 \times \frac{\pi}{2} + r_2 . \alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 + r_2) + x - \frac{(r_1 + r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

Substituting the value of $\alpha = \frac{r_1 + r_2}{x}$

$$L = \pi(r_1 + r_2) + \frac{2(r_1 + r_2)}{x} \times (r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{2(r_1 + r_2)^2}{x} + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x} \qquad \text{...(In terms of pulley radii)}$$

$$= \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x} \qquad \text{...(In terms of pulley diameters)}$$

It may be noted that the above expression is a function of ($r1$ + $r2$). It is thus obvious that if sum of the radii of the two pulleys be constant, then length of the belt required will also remain constant, provided the distance between centres of the pulleys remain unchanged.

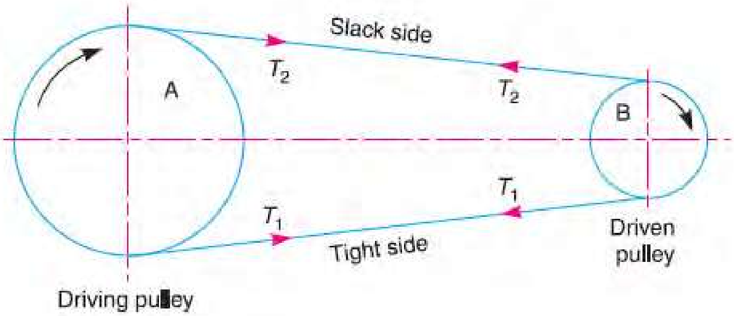
Power transmitted by a Belt

Fig. shows the driving pulley (or driver) *A* and the driven pulley (or follower) *B*. We have already discussed that the driving pulley pulls the belt from one side and delivers the same to the other side. It is thus obvious that the tension on the former side (*i.e.* tight side) will be greater than the latter side (*i.e.* slack side) as shown in Fig.

Let T_1 and T_2 = Tensions in the tight and slack side of the belt respectively in newtons,

r_1 and r_2 = Radii of the driver and follower respectively, and

v = Velocity of the belt in m/s.



The effective turning (driving) force at the circumference of the follower is the difference between the two tensions (*i.e.* $T_1 - T_2$).

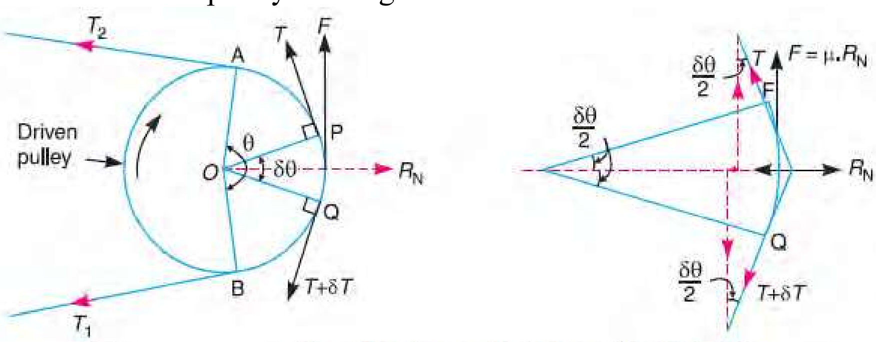
$\therefore$ Work done per second = $(T_1 - T_2)\, v$ N-m/s

and power transmitted, $P = (T_1 - T_2)\, v$ W ...($\because$ 1 N-m/s = 1 W)

A little consideration will show that the torque exerted on the driving pulley is $(T_1 - T_2)\, r_1$. Similarly, the torque exerted on the driven pulley *i.e.* follower is $(T_1 - T_2)\, r_2$.

Ratio of Driving Tensions for Flat Belt Drive

Consider a driven pulley rotating in the clockwise direction as shown in Fig.



Let T_1 = Tension in the belt on the tight side,

T_2 = Tension in the belt on the slack side, and

θ = Angle of contact in radians (*i.e.* angle subtended by the arc *AB*, along which the belt touches the pulley at the centre).

Now consider a small portion of the belt *PQ*, subtending an angle $\delta\theta$ at the centre of the pulley as shown in Fig. 11.15. The belt *PQ* is in equilibrium under the following forces :

1. Tension T in the belt at P,
2. Tension $(T + \delta T)$ in the belt at Q,
3. Normal reaction R_N, and
4. Frictional force, $F = \mu \times R_N$, where μ is the coefficient of friction between the belt and pulley.

Resolving all the forces horizontally and equating the same,

$$R_N = (T + \delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2} \qquad ...(i)$$

Since the angle $\delta\theta$ is very small, therefore putting $\sin \delta\theta / 2 = \delta\theta / 2$ in equation (*i*),

$$R_N = (T + \delta T)\frac{\delta\theta}{2} + T \times \frac{\delta\theta}{2} = \frac{T.\delta\theta}{2} + \frac{\delta T.\delta\theta}{2} + \frac{T.\delta\theta}{2} = T.\delta\theta \qquad ...(ii)$$

$$...\left(\text{Neglecting } \frac{\delta T.\delta\theta}{2}\right)$$

Now resolving the forces vertically, we have

$$\mu \times R_N = (T + \delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2} \qquad ...(iii)$$

Since the angle $\delta\theta$ is very small, therefore putting $\cos \delta\theta / 2 = 1$ in equation (*iii*),

$$\mu \times R_N = T + \delta T - T = \delta T \quad \text{or} \quad R_N = \frac{\delta T}{\mu} \qquad ...(iv)$$

Equating the values of R_N from equations (*ii*) and (*iv*),

$$T.\delta\theta = \frac{\delta T}{\mu} \quad \text{or} \quad \frac{\delta T}{T} = \mu.\delta\theta$$

Integrating both sides between the limits T_2 and T_1 and from 0 to θ respectively,

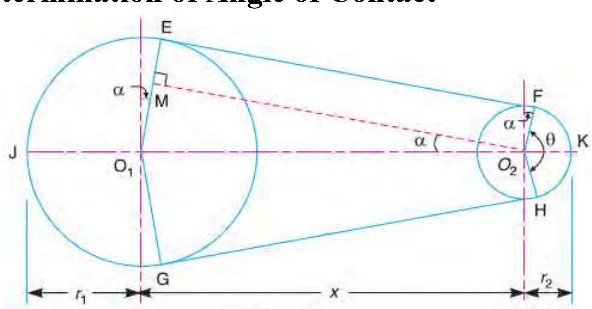
i.e. $$\int_{T_2}^{T_1} \frac{\delta T}{T} = \mu \int_0^{\theta} \delta\theta \quad \text{or} \quad \log_e\left(\frac{T_1}{T_2}\right) = \mu.\theta \quad \text{or} \quad \frac{T_1}{T_2} = e^{\mu.\theta}$$

Equation (*v*) can be expressed in terms of corresponding logarithm to the base 10, *i.e.*

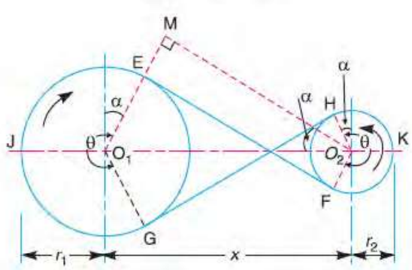
$$2.3\log\left(\frac{T_1}{T_2}\right) = \mu.\theta$$

The above expression gives the relation between the tight side and slack side tensions, in terms of coefficient of friction and the angle of contact.

Determination of Angle of Contact



(*a*) Open belt drive.



(*b*) Crossed belt drive.

When the two pulleys of different diameters are connected by means of an open belt as shown in Fig. (*a*), then the angle of contact or lap (θ) at the smaller pulley must be taken into consideration.

Let r_1 = Radius of larger pulley,

r_2 = Radius of smaller pulley, and

x = Distance between centres of two pulleys (*i.e.* $O_1 O_2$).

$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E - ME}{O_1 O_2} = \frac{r_1 - r_2}{x} \qquad ...(\because ME = O_2 F = r_2)$$

∴ Angle of contact or lap,

$$\theta = (180° - 2\alpha)\frac{\pi}{180} \text{ rad}$$

A little consideration will show that when the two pulleys are connected by means of a crossed belt as shown in Fig. (*b*), then the angle of contact or lap (θ) on both the pulleys is same

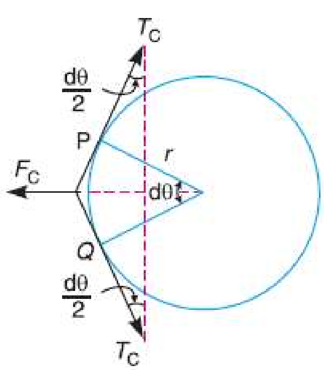
$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E + ME}{O_1 O_2} = \frac{r_1 + r_2}{x}$$

∴ Angle of contact or lap, $\theta = (180° + 2\alpha)\frac{\pi}{180}$ rad

Centrifugal Tension

Since the belt continuously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both, tight as well as the slack sides. The tension caused by centrifugal force is called ***centrifugal tension***. At lower belt speeds (less than 10 m/s), the centrifugal tension is very small, but at higher belt speeds (more than 10 m/s), its effect is considerable and thus should be taken into account.

Consider a small portion *PQ* of the belt subtending an angle $d\theta$ the centre of the pulley as shown in Fig.



Let m = Mass of the belt per unit length in kg,

v = Linear velocity of the belt in m/s,

r = Radius of the pulley over which the belt runs in metres, and

T_C = Centrifugal tension acting tangentially at P and Q in newtons.

We know that length of the belt PQ

$$= r.\,d\theta$$

and mass of the belt PQ $= m.\,r.\,d\theta$

∴ Centrifugal force acting on the belt PQ,

$$F_C = (m.r.d\theta)\frac{v^2}{r} = m.d\theta.v^2$$

The centrifugal tension T_C acting tangentially at P and Q keeps the belt in equilibrium.

Now resolving the forces (*i.e.* centrifugal force and centrifugal tension) horizontally and equating the same, we have

$$T_C \sin\left(\frac{d\theta}{2}\right) + T_C \sin\left(\frac{d\theta}{2}\right) = F_C = m.d\theta.v^2$$

Since the angle $d\theta$ is very small, therefore, putting $\sin\left(\frac{d\theta}{2}\right) = \frac{d\theta}{2}$, in the above expression,

$$2T_C\left(\frac{d\theta}{2}\right) = m.d\theta.v^2 \quad \text{or} \quad T_C = m\,.\,v^2$$

When the centrifugal tension is taken into account, then total tension in the tight side,

$$T_{t1} = T_1 + T_C$$

and total tension in the slack side,

$$T_{t2} = T_2 + T_C$$

Power transmitted, $P = (T_{t1} - T_{t2})\,v$ ...(in watts)

$= [(T_1 + T_C) - (T_2 + T_C)]\,v = (T_1 - T_2)\,v$ ...(same as before)

Thus we see that centrifugal tension has no effect on the power transmitted.

The ratio of driving tensions may also be written as

$$2.3\log\left(\frac{T_{t1} - T_C}{T_{t2} - T_C}\right) = \mu.\theta$$

where T_{t1} = Maximum or total tension in the belt.

Maximum Tension in the Belt

A little consideration will show that the maximum tension in the belt (T) is equal to the total tension in the tight side of the belt (T_{t1}).

Let σ = Maximum safe stress in N/mm²,

b = Width of the belt in mm, and

t = Thickness of the belt in mm.

We know that maximum tension in the belt,

T = Maximum stress × cross-sectional area of belt = $\sigma.\,b.\,t$

When centrifugal tension is neglected, then

T (or T_{t1}) = T_1, *i.e.* Tension in the tight side of the belt

and when centrifugal tension is considered, then

T (or T_{t1}) = $T_1 + T_C$

Condition for the Transmission of Maximum Power

We know that power transmitted by a belt,

$$P = (T_1 - T_2)\,v \qquad ...(i)$$

where T_1 = Tension in the tight side of the belt in newtons,

T_2 = Tension in the slack side of the belt in newtons, and

v = Velocity of the belt in m/s.

From Art. 11.14, we have also seen that the ratio of driving tensions is

$$\frac{T_1}{T_2} = e^{\mu.\theta} \quad \text{or} \quad T_2 = \frac{T_1}{e^{\mu.\theta}} \qquad ...(ii)$$

Substituting the value of T_2 in equation (*i*),

$$P = \left(T_1 - \frac{T_1}{e^{\mu.\theta}}\right)v = T_1\left(1 - \frac{1}{e^{\mu.\theta}}\right)v = T_1.v.C \qquad ...(iii)$$

where

$$C = 1 - \frac{1}{e^{\mu.\theta}}$$

We know that $T_1 = T - T_C$

where T = Maximum tension to which the belt can be subjected in newtons, and

T_C = Centrifugal tension in newtons.

Substituting the value of T_1 in equation (*iii*),

$$P = (T - T_C)\,v.C$$

$$= (T - m.v^2)\,v.C = (T.v - m\,v^3)\,C \qquad \text{... (Substituting } T_C = m.\,v^2\text{)}$$

For maximum power, differentiate the above expression with respect to v and equate to zero, *i.e.*

$$\frac{dP}{dv} = 0 \qquad \text{or} \qquad \frac{d}{dv}(T.v - mv^3)\,C = 0$$

$$\therefore \quad T - 3\,m\,.\,v^2 = 0$$

or

$$T - 3\,T_C = 0 \quad \text{or} \quad T = 3\,T_C \qquad ...(iv)$$

It shows that when the power transmitted is maximum, 1/3rd of the maximum tension is absorbed as centrifugal tension.

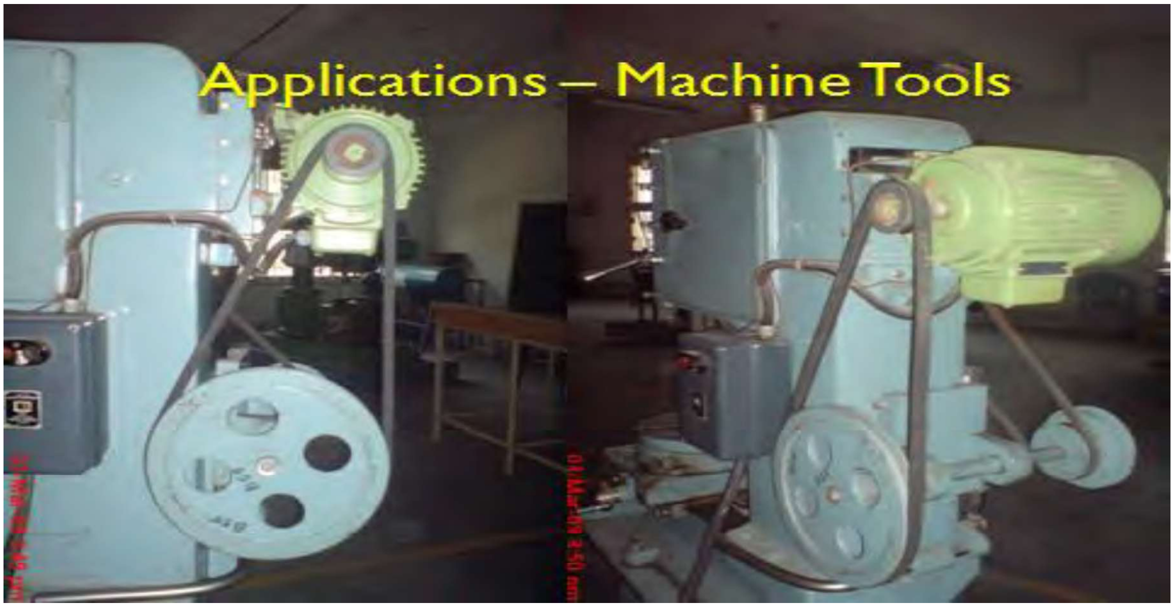
We know that $T_1 = T - T_C$ and for maximum power, $T_C = \dfrac{T}{3}$.

$$T_1 = T - \frac{T}{3} = \frac{2T}{3}$$

From equation (*iv*), the velocity of the belt for the maximum power,

$$v = \sqrt{\frac{T}{3m}}$$

Applications:



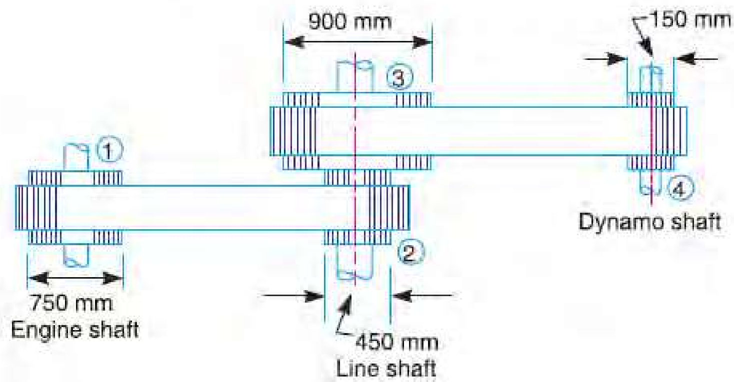
Exercise Problems:

1) An engine, running at 150 r.p.m., drives a line shaft by means of a belt. The engine pulley is 750 mm diameter and the pulley on the line shaft being 450 mm. A 900 mm diameter pulley on the line shaft drives a 150 mm diameter pulley keyed to a dynamo shaft. Find the speed of the dynamo shaft, when 1. there is no slip, and 2. there is a slip of 2% at each drive.

Solution:

Given : N_1 = 150 r.p.m. ; d_1 = 750 mm ; d_2 = 450 mm ; d_3 = 900 mm ; d_4 = 150 mm

Let N_4 = Speed of the dynamo shaft .



1. When there is no slip

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \quad \text{or} \quad \frac{N_4}{150} = \frac{750 \times 900}{450 \times 150} = 10$$

$$N_4 = 150 \times 10 = 1500 \text{ r.p.m.}$$

2. When there is a slip of 2% at each drive

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\frac{N_4}{150} = \frac{750 \times 900}{450 \times 150}\left(1 - \frac{2}{100}\right)\left(1 - \frac{2}{100}\right) = 9.6$$

$$N_4 = 150 \times 9.6 = 1440 \text{ r.p.m.}$$

2) Find the power transmitted by a belt running over a pulley of 600 mm diameter at 200 r.p.m. The coefficient of friction between the belt and the pulley is 0.25, angle of lap 160° and maximum tension in the belt is 2500 N.

Solution:

Given: d = 600 mm = 0.6 m ; N = 200 r.p.m. ; μ = 0.25 ; θ= 160° = 160 × π/ 180 = 2.793 rad ; T_1 = 2500 N

We know that velocity of the belt,

$$v = \frac{\pi d . N}{60} = \frac{\pi \times 0.6 \times 200}{60} = 6.284 \text{ m/s}$$

Let T_2 = Tension in the slack side of the belt.

We know that $$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu . \theta = 0.25 \times 2.793 = 0.6982$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.6982}{2.3} = 0.3036$$

$$\frac{T_1}{T_2} = 2.01 \quad \text{...(Taking antilog of 0.3036)}$$

$$T_2 = \frac{T_1}{2.01} = \frac{2500}{2.01} = 1244 \text{ N}$$

We know that power transmitted by the belt,

$$P = (T_1 - T_2)\, v = (2500 - 1244)\, 6.284 = 7890 \text{ W}$$
$$= 7.89 \text{ kW}$$

3) A casting weighing 9 kN hangs freely from a rope which makes 2.5 turns round a drum of 300 mm diameter revolving at 20 r.p.m. The other end of the rope is pulled by a man. The coefficient of friction is 0.25. Determine **1.** The force required by the man, and **2.** The power to raise the casting.

Solution:

Given : $W = T_1 = 9$ kN = 9000 N ; $d = 300$ mm = 0.3 m ; $N = 20$ r.p.m. ; $\mu = 0.25$

1. *Force required by the man*

Let T_2 = Force required by the man.

Since the rope makes 2.5 turns round the drum, therefore angle of contact,

$\theta = 2.5 \times 2\,\pi = 5\,\pi$ rad

$$2.3\log\left(\frac{T_1}{T_2}\right) = \mu\,.\,\theta = 0.25 \times 5\pi = 3.9275$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{3.9275}{2.3} = 1.71 \quad \text{or} \quad \frac{T_1}{T_2} = 51$$

...(Taking antilog of 1.71)

$$T_2 = \frac{T_1}{51} = \frac{9000}{51} = 176.47 \text{ N}$$

2. *Power to raise the casting*

We know that velocity of the rope,

$$v = \frac{\pi\, d\,.N}{60} = \frac{\pi \times 0.3 \times 20}{60} = 0.3142 \text{ m/s}$$

Power to raise the casting,

$$P = (T_1 - T_2)\, v = (9000 - 176.47)\, 0.3142 = 2772 \text{ W}$$
$$= 2.772 \text{ kW}$$

4) Two pulleys, one 450 mm diameter and the other 200 mm diameter are on parallel shafts 1.95 m apart and are connected by a crossed belt. Find the length of the belt required and the angle of contact between the belt and each pulley. What power can be transmitted by the belt when the larger pulley rotates at 200 rev/min, if the maximum permissible tension in the belt is 1 kN, and the coefficient of friction between the belt and pulley is 0.25 ?

Solution:

Given : $d_1 = 450$ mm = 0.45 m or $r_1 = 0.225$ m ; $d_2 = 200$ mm = 0.2 m or $r_2 = 0.1$ m ; $x = 1.95$ m ; $N_1 = 200$ r.p.m. ; $T_1 = 1$ kN = 1000 N ; $\mu = 0.25$

We know that speed of the belt,

$$v = \frac{\pi d_1 . N_1}{60} = \frac{\pi \times 0.45 \times 200}{60} = 4.714 \text{ m/s}$$

We know that length of the crossed belt,

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(0.225 + 0.1) + 2 \times 1.95 + \frac{(0.225 + 0.1)^2}{1.95} = 4.975\,\text{m}$$

Angle of contact between the belt and each pulley

Let θ = Angle of contact between the belt and each pulley.

We know that for a crossed belt drive,

$$\sin\alpha = \frac{r_1 + r_2}{x} = \frac{0.225 + 0.1}{1.95} = 0.1667 \quad \text{or} \quad \alpha = 9.6°$$

$$\theta = 180° + 2\,\alpha = 180° + 2 \times 9.6° = 199.2°$$

$$= 199.2 \times \frac{\pi}{180} = 3.477 \text{ rad}$$

Power transmitted

Let T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 3.477 = 0.8692$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.8692}{2.3} = 0.378 \quad \text{or} \quad \frac{T_1}{T_2} = 2.387 \qquad \text{...(Taking antilog of 0.378)}$$

$$\therefore \quad T_2 = \frac{T_1}{2.387} = \frac{1000}{2.387} = 419 \text{ N}$$

We know that power transmitted,

$$P = (T_1 - T_2)\,v = (1000 - 419)\,4.714 = 2740 \text{ W} = 2.74 \text{ kW}$$

5) A shaft rotating at 200 r.p.m. drives another shaft at 300 r.p.m. and transmits 6 kW through a belt. The belt is 100 mm wide and 10 mm thick. The distance between the shafts is 4m. The smaller pulley is 0.5 m in diameter. Calculate the stress in the belt, if it is **1.** an open belt drive, and **2.** a cross belt drive. Take μ =0.3.

Solution:

Given : N_1 = 200 r.p.m. ; N_2 = 300 r.p.m. ; P = 6 kW = 6 × 10^3 W ; b = 100 mm ; t = 10 mm ; x = 4 m ; d_2 = 0.5 m ; μ = 0.3

Let σ= Stress in the belt.

1. *Stress in the belt for an open belt drive*

First of all, let us find out the diameter of larger pulley (d_1). We know that

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{or} \quad d_1 = \frac{N_2.d_2}{N_1} = \frac{300 \times 0.5}{200} = 0.75\,\text{m}$$

and velocity of the belt,

$$v = \frac{\pi\, d_2.N_2}{60} = \frac{\pi \times 0.5 \times 300}{60} = 7.855 \text{ m/s}$$

Now let us find the angle of contact on the smaller pulley. We know that, for an open belt drive,

$$\sin\alpha = \frac{r_1 - r_2}{x} = \frac{d_1 - d_2}{2x} = \frac{0.75 - 0.5}{2 \times 4} = 0.03125 \quad \text{or} \quad \alpha = 1.8°$$

$\therefore$ Angle of contact, $\theta = 180° - 2\alpha = 180 - 2 \times 1.8 = 176.4°$

$= 176.4 \times \pi / 180 = 3.08$ rad

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu . \theta = 0.3 \times 3.08 = 0.924$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.924}{2.3} = 0.4017 \quad \text{or} \quad \frac{T_1}{T_2} = 2.52$$

We also know that power transmitted (P),

$$6 \times 10^3 = (T_1 - T_2)\, v = (T_1 - T_2)\, 7.855$$

$$\therefore \quad T_1 - T_2 = 6 \times 10^3 / 7.855 = 764 \text{ N}$$

By solving the above two equations

T1= 1267 N and T2= 503 N

We know that maximum tension in the belt (T_1),

$$1267 = \sigma . b . t = \sigma \times 100 \times 10 = 1000\,\sigma$$

$$\sigma = 1267 / 1000 = 1.267 \text{ N/mm}^2 = 1.267 \text{ MPa}$$

...[$\because$ 1 MPa = 1 MN/m^2 = 1 N/mm^2]

Stress in the belt for a cross belt drive

We know that for a cross belt drive,

$$\sin\alpha = \frac{r_1 + r_2}{x} = \frac{d_1 + d_2}{2x} = \frac{0.75 + 0.5}{2 \times 4} = 0.1562 \quad \text{or} \quad \alpha = 9°$$

$\therefore$ Angle of contact, $\theta = 180° + 2\alpha = 180 + 2 \times 9 = 198°$

$= 198 \times \pi / 180 = 3.456$ rad

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu . \theta = 0.3 \times 3.456 = 1.0368$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{1.0368}{2.3} = 0.4508 \quad \text{or} \quad \frac{T_1}{T_2} = 2.82$$

By solving the above equations

T_1 = 1184 N and T_2 = 420 N

We know that maximum tension in the belt (T_1),

$$1184 = \sigma . b . t = \sigma \times 100 \times 10 = 1000\,\sigma$$

$$\sigma = 1184 / 1000 = 1.184 \text{ N/mm}^2 = 1.184 \text{ MPa}$$

6) Determine the width of a 9.75 mm thick leather belt required to transmit 15 kW from a motor running at 900 r.p.m. The diameter of the driving pulley of the motor is 300 mm. The driven pulley runs at 300 r.p.m. and the distance between the centre of two pulleys is 3 metres. The density of the leather is 1000 kg/m^3. The maximum allowable stress in the leather is 2.5 MPa. The coefficient of friction between the leather and pulley is 0.3. Assume open belt drive and neglect the sag and slip of the belt.

Solution:

Given: t = 9.75 mm = 9.75×10^{-3} m ; P = 15 kW = 15×10^3 W ; N_1 = 900 r.p.m. ; d_1 = 300 mm = 0.3 m ; N_2 = 300 r.p.m. ; x = 3m ; ρ= 1000 kg/m^3 ; σ= 2.5 MPa = 2.5×10^6 N/m^2 ; μ = 0.3

First of all, let us find out the diameter of the driven pulley (d_2). We know that

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \text{ or } d_2 = \frac{N_1 \times d_1}{N_2} = \frac{900 \times 0.3}{300} = 0.9 \text{ m}$$

velocity of the belt, $$v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 0.3 \times 900}{60} = 14.14 \text{ m/s}$$

For an open belt drive,

$$\sin\alpha = \frac{r_2 - r_1}{x} = \frac{d_2 - d_1}{2x} = \frac{0.9 - 0.3}{2 \times 3} = 0.1 \quad ...(\because d_2 > d_1)$$

$$\alpha = 5.74°$$

∴ Angle of lap, $\theta = 180° - 2\alpha = 180 - 2 \times 5.74 = 168.52°$

$= 168.52 \times \pi / 180 = 2.94$ rad

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 2.94 = 0.882$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.882}{2.3} = 0.3835 \text{ or } \frac{T_1}{T_2} = 2.42$$

We also know that power transmitted (P),

$$15 \times 10^3 = (T_1 - T_2)\, v = (T_1 - T_2)\, 14.14$$

$$\therefore \quad T_1 - T_2 = 15 \times 10^3 / 14.14 = 1060 \text{ N}$$

On solving above two equations we get T1= 1806 N

Let b = Width of the belt in metres.

We know that mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b.t.l.\rho$$

$$= b \times 9.75 \times 10^{-3} \times 1 \times 1000 = 9.75\, b \text{ kg}$$

∴ Centrifugal tension,

$$T_C = m.v^2 = 9.75\, b\, (14.14)^2 = 1950\ b \text{ N}$$

Maximum tension in the belt,

$$T = \sigma.\, b.\, t = 2.5 \times 10^6 \times b \times 9.75 \times 10^{-3} = 24\,400\, b \text{ N}$$

We know that $T = T_1 + T_C$ or $T - T_C = T_1$

$$24\,400\, b - 1950\, b = 1806 \text{ or } 22\,450\, b = 1806$$

$$\therefore \quad b = 1806 / 22\,450 = 0.080 \text{ m} = 80 \text{ mm}$$