SANT LONGOWAL INSTITUTE OF ENGINEERING AND TECHNOLOGY

Deemed- To- be University, under MoE

Course Material

Subject: Engineering Mechanics

Subject Code: ESME-501

DEPARTMENT OF MECHANICAL ENGINEERING

Prepared by

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For UG Program

Title of the course : Engineering Mechanics
Subject Code : ESME-501

L	T	P	Credits	Weekly Load(hrs)
3	1	0	4	4

COURSE OUTCOMES:
After successful completion of course, the students should be able to
CO1: Understand the importance of mechanics in the context of engineering.
CO2: Analyze the various forces acting on engineering components
CO3: Apply the different principles to study the motion of a body, and concept of relative velocity and acceleration.
CO4: Analyse various forces acting on elements of truss
CO5: Identify the basic elements of a mechanical system and write their constitutive equations.

Pre-requisite knowledge:

CO/PO Mapping: (Strong(3) / Medium(2) / Weak(1) indicates strength of correlation):															
Cos	Programme Outcomes (POs)												Programme Specific Outcomes		
	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO 1	PSO 2	PSO 3
CO1	3	3	3	3	2	2	1	1	1	1	-	1	2	1	2
CO2	3	3	3	3	2	2	1	1	1	1	-	1	2	1	2
CO3	3	3	3	3	2	2	1	1	1	1	-	1	2	1	2
CO4	3	3	3	3	2	2	1	1	1	1	-	1	2	1	2
CO5	3	3	3	3	2	2	1	1	1	1	1	1	2	1	2
Avg.	3	3	3	3	2	2	1	1	1	1	1	1	2	2	2

Unit	Main Topics	Course outlines	Lecture(s)
Unit-1	**Fundamental of Mechanics**	Mechanics and its relevance, Fundamental concept of mechanics and applied mechanics, idealization of mechanics, Basic dimensions and units of measurements, concept of rigid bodies, Laws of Mechanics	**03**
	Laws for Forces	Control Scalars and Vectors, Vector operations, Vector addition of forces, Force and its effects, characteristics of force vector, Bow's notation	**04**

		Force systems: Coplanar and Space force systems. Coplanar concurrent and non-concurrent forces. Free body diagrams	
	Resultant and components of forces	Concept of equilibrium; Parallelogram law of forces, equilibrium of two forces; super position and transmissibility of forces, Newton's third law, triangle law of forces, different cases of concurrent, coplanar two forces systems, extension of parallelogram law and triangle law to many forces acting at one point.	**04**
	Polygon law of forces	Triangle law to many forces acting at one point - polygon law of forces, method of resolution into orthogonal components for finding the resultant, graphical methods, special case of three concurrent, coplanar forces, Lami's theorem	**04**
	Moments & Couples	Concept of moment, Varignon's theorem, Principle of moments, Moment of forces about a specified axis, concept of couple - properties and effect, Moment of couple, Movement of force on rigid body, Resultant of force and couple system, Reduction of force and couple system, Parallel forces - like and unlike parallel forces, calculation of their resultant.	**04**
	Trusses	Simple trusses, analysis of simple truss, Method of Joints, Method of sections	**04**
Unit-2	**Friction**	Concept of friction, Characteristics of Dry friction, Laws of Coulomb friction, limiting friction, coefficient of friction; sliding friction and rolling friction, Belt friction, Ladder friction.	**04**
	Centre of gravity and Moment of Inertia	Concept of gravity, gravitational force, centroid and centre of gravity, centroid for regular lamina and centre of gravity for regular solids. Position of centre of gravity of compound bodies and centroid of composite area. CG of bodies with portions removed, Moment of Inertia: First and second moment of area; Radius of gyration, Moment of inertia of simple and composite bodies.	**06**
	Simple Lifting Machines	Concept of machine, mechanical advantage, velocity ratio and efficiency of a machine, their relationship, law of machine, Simple machines: Wheel and axle, pulley systems, Simple screw jacks	**03**

	Kinetics of a particle	Types of motion, linear motion with uniform velocity, uniform & varying acceleration, motion under gravity, motion of projectiles, concept of relative and resultant velocity. Newton's laws of motion, equation of motion for system of particles, D' Alembet's Principle, Motion of connecting bodies. Concept of momentum, Impulse momentum, Conservation of momentum and energy, Principle of work and energy	**06**
	Kinetics of a rigid body	Introduction, Equation of motion for a rigid body, Angular Motion of Rigid Bodies, D'Alembert's principle applied to bodies having linear and angular motion. Equation of dynamic equilibrium, Maximum acceleration and retardation of vehicles running on inclined planes.	06

Total=48

Recommended Books:

1. J. L. Mariam & L. G. Kraige , Engineering Mechanics. John Wiley & Sons
2. R. C. Hibbeler, Engineering Mechanics (Static & Dynamics), Prentice Hall
3. Beer & Johnston, Engineering Mechanics (Static & Dynamics), McGraw Hill
4. Boresi&Schimidt, Engineering Mechanics (Static & Dynamics), Cengage Learning
5. R. K. Rajput, Engineering Mechanics, Dhanpat Rai Publication, New Delhi
6. S. Rajshekharan, Engineering Mechanics, VIkas Publishing House , New Delhi

DISCLAIMER

CHAPTER 1 FUNDAMENTALS OF MECHANICS

Course content
Mechanics and its relevance, Fundamental concept of mechanics and applied mechanics, idealization of mechanics, Basic dimensions and units of measurements, concept of rigid bodies, Laws of Mechanics

1 Introduction

Mechanics is a branch of physics that deals with the study of motion and forces acting on objects. It plays a crucial role in engineering applications, including the design and analysis of machines, structures, and vehicles.

1.1 FUNDAMENTAL CONCEPT OF MECHANICS AND APPLIED MECHANICS

Engineering mechanics can be broadly classified into:

i. **Theoretical Mechanics (Classical Mechanics)**: Deals with the motion of bodies under the influence of forces without considering the effect of engineering applications.
ii. **Applied Mechanics (Engineering Mechanics)**: Focuses on practical applications, analyzing forces acting on different structures and materials to ensure their proper functioning.

Real Life Examples:

The working of a car suspension system, where applied mechanics helps in designing a shock absorber to minimize vibrations.

The stability of a bridge, where forces acting on the structure are analyzed to ensure safety.

1.2 IDEALIZATION IN MECHANICS

To simplify the analysis of mechanical problems, certain assumptions or idealizations are made:

i. **Particle:** A body whose dimensions are negligible compared to the problem's scale. Example: A satellite treated as a particle in orbit calculations.

ii. **Rigid Body:** A body that does not deform under the influence of forces. Example: A steel beam in construction.

iii. **Point Mass:** When an object's size is negligible compared to the problem's scale. Example: Earth is treated as a point mass while studying planetary motion.

iv. **Continuum:** In Engineering Mechanics, the concept of Continuum refers to an idealized model of a material body where the matter is continuously distributed and fills the entire region of space it occupies, without any gaps or voids. This assumption is fundamental in mechanics and helps simplify the analysis of materials and structures.

1.3 BASIC DIMENSIONS AND UNITS OF MEASUREMENT

Fundamental Dimensions**:** In mechanics, the most commonly used **fundamental dimensions** are:

Quantity	**Dimension Symbol**	**Description**
Length	L	Measures distance, displacement, or size of an object.
Mass	M	Represents the quantity of matter in a body.
Time	T	Defines the duration of an event.
Temperature	Θ	Measures the thermal state of a body.

All other physical quantities in mechanics are derived from these fundamental dimensions.

1.3.1 DERIVED QUANTITIES AND THEIR DIMENSIONS

Some key quantities derived in engineering mechanics are:

Derived Quantity	Expression	Dimension
Area	L×L	L^2
Volume	L×L×L	L^3
Velocity	Displacement/Time	M^0LT^{-1}
Acceleration	Velocity/Time	M^0LT^{-2}
Force (Newton)	Mass ×Acceleration	MLT^{-2}
Pressure	Force/Area	$ML^{-1}T^{-2}$
Work / Energy	Force× Distance	ML^2T^{-2}
Power	Work/Time	ML^2T^{-3}

1.3.2 SYSTEMS OF UNITS

Different unit systems are used in engineering mechanics:

(a) International System of Units (SI): Most widely used system in engineering and science.

Quantity	SI Unit	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s

Quantity	SI Unit	Symbol
Force	Newton	$N=kg{\cdot}m/s^2$
Work/Energy	Joule	$J=N{\cdot}m$
Power	Watt	$W=J/s$

(b) FPS System (Foot-Pound-Second)**:** Commonly used in the U.S.

Quantity	FPS Unit	Symbol
Length	Foot	ft
Mass	Slug	slug
Time	Second	s
Force	Pound-force	lbf

(c) CGS System (Centimeter-Gram-Second)**:** Used for smaller-scale applications.

Quantity	CGS Unit	Symbol
Length	Centimeter	cm
Mass	Gram	g
Force	Dyne	dyne

1.4 CONCEPT OF RIGID BODIES

A **rigid body** is an idealized solid object that does not deform under force.

Example: A metal rod under compression remains unchanged in shape, whereas a rubber sheet deforms.

In real-life applications, rigid body assumptions simplify complex calculations while designing structures like cranes, airplanes, and gears.

Real-Life Example:

A hammer is treated as a rigid body when designing its impact force.

A wind turbine's blades are considered rigid for aerodynamic analysis.

Conclusion

The study of mechanics is fundamental to engineering applications, ensuring the safety and efficiency of mechanical systems. Understanding the basic concepts, including idealizations and unit systems, helps in designing practical solutions for real-world problems.

CHAPTER 2 LAWS OF FORCES

Course content
Control Scalars and Vectors, Vector operations, Vector addition of forces, Force and its effects, characteristics of force vector, Bow's notation, Force systems: Coplanar and Space force systems. Coplanar concurrent and non-concurrent forces. Free body diagrams

2.1 INTRODUCTION

In Engineering Mechanics, forces are the agents that cause bodies to move, deform, or remain in equilibrium. A proper understanding of forces and their effects is essential for analyzing and designing mechanical systems. This chapter introduces the basic nature of forces, the fundamental laws governing them, and the methods used to analyze force systems.

2.2 BASIC CONCEPTS OF FORCE

Force is a vector quantity characterized by both magnitude and direction. It is measured in units such as Newtons (N) in the SI system. Forces can cause acceleration (change in velocity) or deformation of a body.

2.3 TYPES OF FORCES

i. **Contact Forces:** Arise from physical interactions (e.g., friction, tension, normal force).
ii. **Non-Contact Forces:** Act at a distance (e.g., gravitational, electromagnetic forces).

2.4 REPRESENTING FORCES

Forces are graphically represented by arrows. The arrow’s length indicates the magnitude, while its direction shows the line of action. When several forces act on a body, their net effect can be determined using vector addition.

2.5 CONCEPT OF SCALARS AND VECTORS

i. **Scalars:** Quantities that have only magnitude (e.g., mass, time, speed, energy).
ii. **Vectors:** Quantities that have both magnitude and direction (e.g., displacement, velocity, force).

2.5.1 VARIOUS VECTOR OPERATIONS

i. **Addition:** Vector sum of two or more vectors.
ii. **Subtraction:** Difference between two vectors.
iii. **Multiplication by a Scalar:** Scaling the magnitude of a vector without changing its direction.
iv. **Dot Product:** Measures the projection of one vector onto another.
v. **Cross Product:** Finds a perpendicular vector to two given vectors.

2.6 VECTOR ADDITION OF FORCES

i. **Triangle Law:** If two forces are represented as two sides of a triangle, their resultant is given by the closing side.
ii. **Parallelogram Law:** If two forces acting at a point are represented by adjacent sides of a parallelogram, their resultant is given by the diagonal.
iii. **Polygon Law of Forces:** Extends the triangle law to any number of forces acting concurrently. If the forces are arranged head-to-tail, they form a polygon; if the forces are in equilibrium, the polygon will be closed.

2.7 CHARACTERISTICS OF FORCE VECTOR

A force vector is defined by:

I. **Magnitude:** The size or intensity of the force (measured in Newtons, N, in the SI system).
II. **Direction:** The line along which the force acts.
III. **Line of Action:** The straight line along which the force vector is applied.
IV. **Point of Application:** The specific point on a body where the force is applied.

2.8 BOW'S NOTATION

Bow's notation is a standardized symbolic method used to represent forces, their directions, and points of application in free-body diagrams (FBDs) and structural analysis. In Bow's notation:

- Arrows represent forces.

- The tail of the arrow indicates the point of application.
- The arrow's head shows the direction, while its length represents the magnitude.
- This notation helps in visualizing and analyzing complex force systems consistently.

2.9 SYSTEM OF FORCES

A **system of forces** refers to a group of forces acting simultaneously on a body. Understanding how these forces interact and affect the body is essential for analyzing equilibrium, stability, and the overall response of structures and mechanisms.

2.9.1 CLASSIFICATION OF FORCE SYSTEMS

i. **Coplanar Force System:** Forces lie in the same plane.
ii. **Space Force System:** Forces act in three-dimensional space.
iii. **Coplanar Concurrent Forces:** Forces in the same plane and meet at a single point.
iv. **Coplanar Non-Concurrent Forces:** Forces in different planes do not meet at a single point.

2.10 LAWS OF MECHANICS

2.10.1 NEWTON'S FIRST LAW (LAW OF INERTIA)

Statement: A body at rest will remain at rest, and a body in motion will continue to move with a constant velocity (i.e., constant speed and direction) unless acted upon by a net external force.

Implication: This law defines inertia, the inherent resistance of any physical object to any change in its state of motion.

2.10.2 NEWTON'S SECOND LAW (LAW OF ACCELERATION)

Statement: The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

Mathematical Formulation: $\sum F = ma$ where: $\sum F$ is the vector sum of all external forces, m is the mass of the object, a is the acceleration of the body.

Implication: This law quantifies the effect of forces and is used to determine how the velocity of an object changes when forces are applied.

2.10.3 NEWTON'S THIRD LAW (ACTION AND REACTION)

Statement: For every action, there is an equal and opposite reaction.

Implication: When one body exerts a force on a second body, the second body simultaneously exerts a force of equal magnitude but in the opposite direction on the first body. This principle is fundamental in understanding interactions between objects.

2.11 FREE BODY DIAGRAMS (FBD)

An FBD is an essential tool for analyzing forces acting on a body. The steps in drawing an FBD are:

1. **Isolate the Body:** Remove the body from its environment.
2. **Identify All Forces:** Include gravitational, applied, frictional, normal, and reaction forces.
3. **Apply Bow's Notation:** Use arrows to represent each force with correct magnitude, direction, and point of application.
4. **Label and Resolve:** Mark distances and angles as needed to resolve forces into components.

2.12 EQUILIBRIUM OF FORCES

A body under the action of coplanar concurrent forces is in equilibrium when the sum of all forces acting on the body is zero, i.e.

$$\sum F=0$$

2.13 SUPERPOSITION AND TRANSMISSIBILITY OF FORCES

i. **Superposition Principle:** The net effect of multiple forces acting on a body is the vector sum of the individual forces.

ii. **Transmissibility of Forces:** A force can be moved along its line of action without altering its external effect on the body. This principle simplifies analysis in FBDs and equilibrium problems.

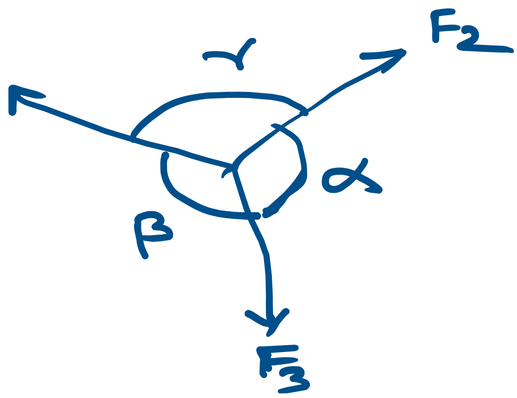
2.14 LAMI'S THEOREM:

For a body in equilibrium under the action of three concurrent, non-collinear forces, Lami's theorem states: when three forces are in equilibrium at a point, each force is proportional to the sine of the angle between the other two forces.

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

→ Prove Sine Law

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

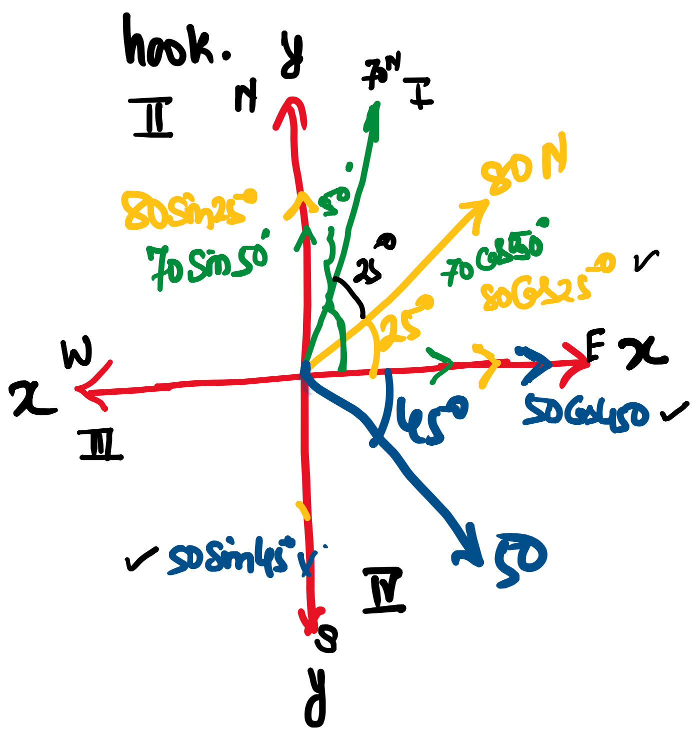


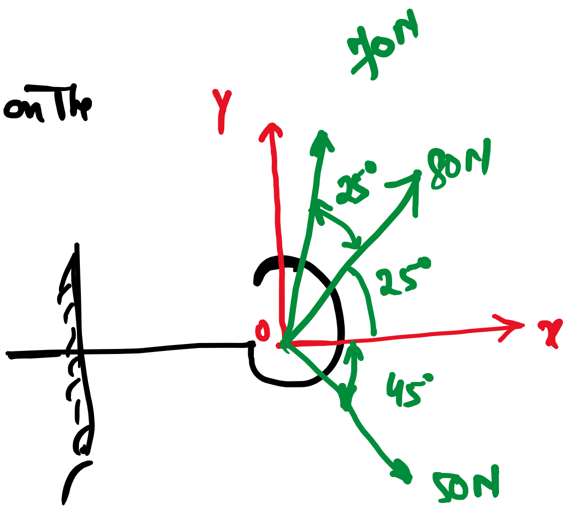
Sample problem

Composition and Resolution of force

F_1, F_2 $F_1 = -F_2$

Q:- Calculate the net resultant on the hook.





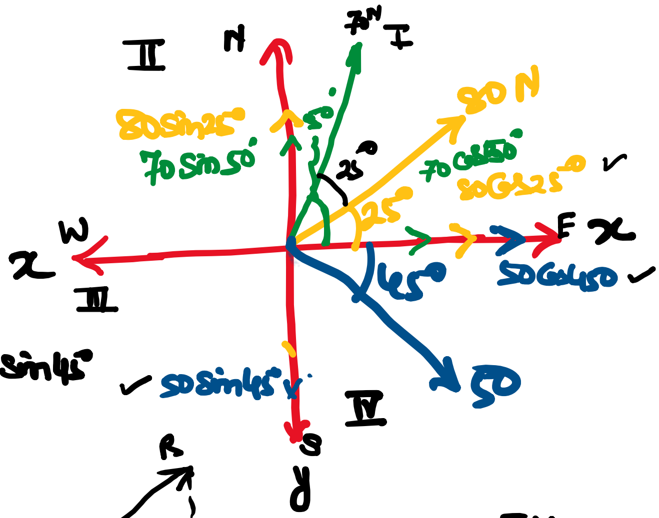
$\Sigma x = \Sigma F_x$ → Net force in x direction

$\Sigma y = \Sigma F_y$ = Net force in y-Direction

$\Sigma X = \Sigma F_x$

$= 70 \cos 50° + 80 \cos 25° + 50 \cos 45°$

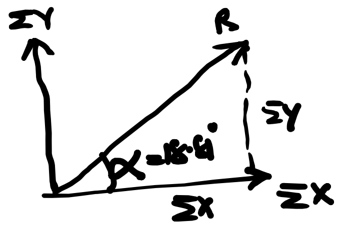
$= 152.86$ N



$\Sigma Y = \Sigma F_y =$

$= 80 \sin 25° + 70 \sin 50° - 50 \sin 45°$

$= 52.07$ N



$|\vec{R}| = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2}$

$= \sqrt{(152.86)^2 + (52.07)^2}$

$= 161.48$ N

$\tan\alpha = \frac{\Sigma Y}{\Sigma X}$

$\alpha = \tan^{-1} \frac{52.07}{152.86}$

$\alpha = 18.81°$

CHAPTER 3 RESULTANT AND COMPONENTS OF FORCES

Course content
Concept of equilibrium; Parallelogram law of forces, equilibrium of two forces; super position and transmissibility of forces, Newton's third law, triangle law of forces, different cases of concurrent, coplanar two forces systems, extension of parallelogram law and triangle law to many forces acting at one point.

3.1 INTRODUCTION

A body is said to be in **equilibrium** when it remains in a state of rest or moves with uniform velocity under the influence of applied forces. In simple terms, equilibrium occurs when the resultant force acting on a body is zero.

3.1.1 TYPES OF EQUILIBRIUM

- **Static Equilibrium**: A body is at rest (e.g., a book lying on a table).
- **Dynamic Equilibrium**: A body moves with uniform velocity (e.g., a car moving at a constant speed on a straight road).

3.2 PRINCIPLES OF EQUILIBRIUM

Though there are many principles of equilibrium, the following three are important from the subject point of view :

1. Two force principle: As per this principle, if a body in equilibrium is acted upon by two forces, then they must be equal, opposite and Collinear.
2. Three force principle: As per this principle, if a body in equilibrium is acted upon by three forces, then the resultant of any two forces must be equal, opposite and collinear with the third force.
3. Four force principle: As per this principle, if a body in equilibrium is acted upon by four forces, then the resultant of any two forces must be equal, opposite and collinear with the resultant of the other two forces.

3.2.1 CONDITIONS FOR EQUILIBRIUM

For a body to be in equilibrium:

1. The sum of all horizontal forces must be zero: $\Sigma x = 0$
2. The sum of all vertical forces must be zero: $\Sigma y = 0$
3. The sum of all moments about any point must be zero: $\Sigma M = 0$

3.3 PARALLELOGRAM LAW OF FORCES

If two forces acting at a point are represented in magnitude and direction by the adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through the common point.

3.4 EQUILIBRIUM OF TWO FORCES

For two forces to be in equilibrium, they must be:

1. Equal in magnitude.
2. Opposite in direction.
3. Collinear.

3.5 SUPERPOSITION AND TRANSMISSIBILITY OF FORCES

3.5.1 SUPERPOSITION PRINCIPLE

The net effect of multiple forces acting on a body is the vector sum of all individual forces.

3.5.2 TRANSMISSIBILITY OF FORCES

A force can be moved along its line of action without changing its external effect on a body.

3.6 NEWTON'S THIRD LAW OF MOTION

3.6.1 STATEMENT

For every action, there is an equal and opposite reaction.

3.6.2 EXAMPLES

1. A person pushing against a wall experiences an equal force in the opposite direction.
2. A rocket propels forward due to the reaction force of expelling gases.

3.7 TRIANGLE LAW OF FORCES

3.7.1 STATEMENT

If two forces acting on a body are represented in magnitude and direction by the two sides of a triangle taken in order, then the closing side represents the resultant force in magnitude and direction.

3.8 CONCURRENT, COPLANAR TWO-FORCE SYSTEMS

3.8.1 CASES

1. **Collinear Forces**: Forces act along the same line (e.g., tension in a rope).
2. **Parallel Forces**: Forces act parallel but may not be collinear (e.g., beam under load).
3. **Non-Parallel Concurrent Forces**: Forces meet at a point but are not parallel.

3.9 EXTENSION TO MULTIPLE FORCES

For more than two forces acting at a point, the **force polygon method** is used.

3.9.1 FORCE POLYGON METHOD

If a set of forces acting at a point forms a closed polygon when drawn in sequence, the forces are in equilibrium.

3.9.2 LAMI'S THEOREM

It states, "If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two." Mathematically, where **P, Q, R** are forces, and **A, B, C** are the angles opposite to the respective forces.

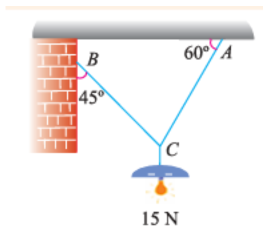
$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

3.9.3 APPLICATIONS IN ENGINEERING

- Structural analysis of bridges and trusses.
- Forces in mechanical linkages.
- Equilibrium of static structures.

Sample Problems

1. An electric light fixture weighting 15 N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal as shown in Figure.



Solution. Given : Weight at C = 15 N

Let T_{AC} = Force in the string AC, and

T_{BC} = Force in the string BC.

The system of forces is shown in Fig. 5.4. From the geometry of the figure, we find that angle between T_{AC} and 15 N is 150° and angle between T_{BC} and 15 N is 135°.

$\therefore \quad \angle ACB = 180° - (45° + 60°) = 75°$

Applying Lami's equation at C,

$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 135°} = \frac{T_{BC}}{\sin 150°}$$

or

$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 45°} = \frac{T_{BC}}{\sin 30°}$$

$$\therefore \quad T_{AC} = \frac{15 \sin 45°}{\sin 75°} = \frac{15 \times 0.707}{0.9659} = 10.98\text{ N} \quad \textbf{Ans.}$$

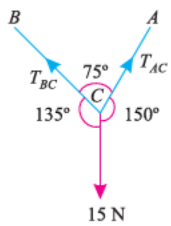
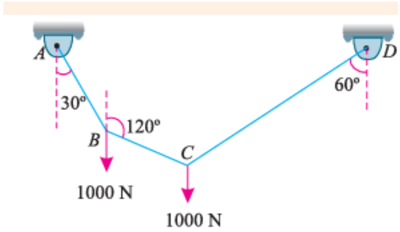


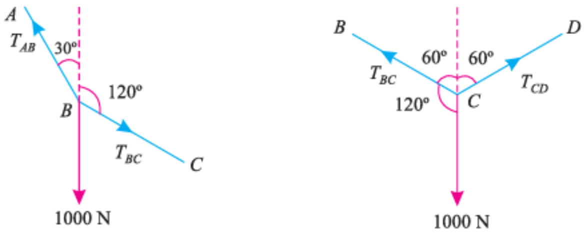
Fig. 5.4.

2. A string ABCD, attached to fixed points A and D has two equal weights of 1000 N attached to it at B and C. The weights rest with the portions AB and CD inclined at angles as shown in Fig. Find the tensions in the portions AB, BC and CD of the string, if the inclination of the portion BC with the vertical is 120°.



Solution. Given : Load at B = Load at C = 1000 N

For the sake of convenience, let us split up the string *ABCD* into two parts. The system of forces at joints *B* and is shown in Fig. 5.6 (*a*) and (*b*).



(*a*) Joint *B* (*b*) Joint *C*

Fig. 5.6.

Let T_{AB} = Tension in the portion *AB* of the string,

T_{BC} = Tension in the portion *BC* of the string, and

T_{CD} = Tension in the portion *CD* of the string.

Applying Lami's equation at joint *B*,

$$\frac{T_{AB}}{\sin 60°} = \frac{T_{BC}}{\sin 150°} = \frac{1000}{\sin 150°}$$

$$\frac{T_{AB}}{\sin 60°} = \frac{T_{BC}}{\sin 30°} = \frac{1000}{\sin 30°} \qquad ...[\because \sin (180° - \theta) = \sin \theta]$$

$$\therefore \qquad T_{AB} = \frac{1000 \sin 60°}{\sin 30°} = \frac{1000 \times 0.866}{0.5} = 1732 \text{ N} \quad \textbf{Ans.}$$

and

$$T_{BC} = \frac{1000 \sin 30°}{\sin 30°} = 1000 \text{ N} \quad \textbf{Ans.}$$

CHAPTER 4 POLYGON LAW OF FORCES

Course content
Triangle law to many forces acting at one point - polygon law of forces, method of resolution into orthogonal components for finding the resultant, graphical methods, special case of three concurrent, coplanar forces, Lami's theorem

4.1 INTRODUCTION

In this chapter we will study the triangle and polygon law of forces along with their application.

4.1.1 TRIANGLE LAW OF FORCES

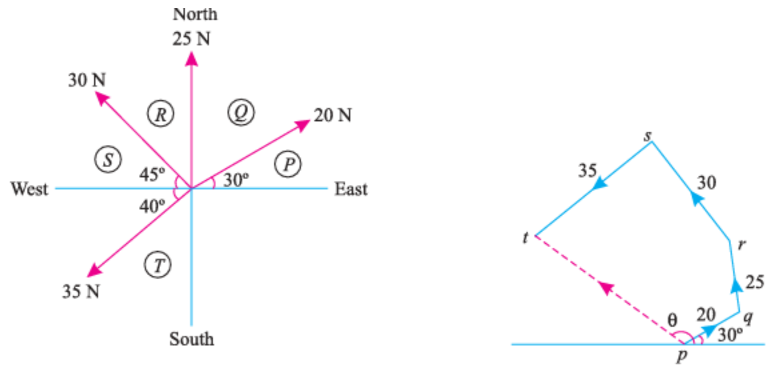
It states, If two forces acting simultaneously on a particle, be represented in magnitude and direction by the two sides of a triangle, taken in order; their resultant may be represented in magnitude and direction by the third side of the triangle, taken in opposite order.

4.1.2 POLYGON LAW OF FORCES

It is an extension of Triangle Law of Forces for more than two forces, which states, "If a number of forces acting simultaneously on a particle, be represented in magnitude and direction, by the sides of a polygon taken in order ; then the resultant of all these forces may be represented, in magnitude and direction, by the closing side of the polygon, taken in opposite order."

Sample problem

1. The following forces act at a point: (i) 20 N inclined at 30° towards North of East. (ii) 25 N towards North. (iii) 30 N towards Northwest and (iv) 35 N inclined at 40° towards South of West. Find the magnitude and direction of the resultant force.
   Solution:
   First of all, name the forces according to Bow's notations as shown in Fig. The 20 N force is named as PQ, 25 N force as QR, 30 N force as RS and 35 N force as ST.



(*a*) Space diagram (*b*) Vector diagram

Fig. 2.9

Now draw the vector diagram for the given system of forces as shown in Fig. (b) and as discussed below:

1. Select some suitable point p and draw pq equal to 20 N to some suitable scale and parallel to the force PQ.

2. Through q, draw qr equal to 25 N to the scale and parallel to the force QR of the space diagram.

3. Now through r, draw rs equal to 30 N to the scale and parallel to the force RS of the space diagram.

4. Similarly, through s, draw st equal to 35 N to the scale and parallel to the force ST of the space diagram.

5. Joint pt, which gives the magnitude as well as direction of the resultant force.

6. By measurement, we find that the magnitude of the resultant force is equal to 45.6 N and acting at an angle of 132° with the horizontal i.e. East–West line.

4.2 RESOLUTION OF A FORCE

The process of splitting up the given force into a number of components, without changing its effect on the body is called resolution of a force. A force is, generally, resolved along two mutually perpendicular directions. In fact, the resolution of a force is the reverse action of the addition of the component vectors

4.2.1 PRINCIPLE OF RESOLUTION

It states, "The algebraic sum of the resolved parts of a no. of forces, in a given direction, is equal to the resolved part of their resultant in the same direction."

4.2.2 METHOD OF RESOLUTION FOR THE RESULTANT FORCE

1. Resolve all the forces horizontally and find the algebraic sum of all the horizontal components (i.e., ∑H).

2. Resolve all the forces vertically and find the algebraic sum of all the vertical components (i.e., ∑V).

3. The resultant R of the given forces will be given by the equation: $R^2 = \sum H^2 + \sum v^2$

4. The resultant force will be inclined at an angle θ, with the horizontal, such that Tan θ= ∑V/∑H

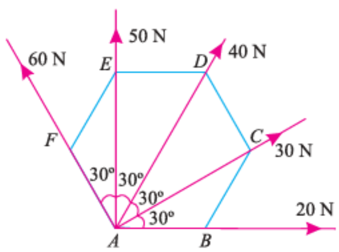
Notes : The value of the angle θ will vary depending upon the values of ∑V and ∑H as discussed below :

1. When ∑V is +ve, the resultant makes an angle between 0° and 180°. But when ∑V is –ve, the resultant makes an angle between 180° and 360°. Component Force Vector

2. When ∑H is +ve, the resultant makes an angle between 0° to 90° or 270° to 360°. But when ∑H is –ve, the resultant makes an angle between 90° to 270°

Sample Problem

1. The forces 20 N, 30 N, 40 N, 50 N and 60 N are acting at one of the angular points of a regular hexagon, towards the other five angular points, taken in order. Find the magnitude and direction of the resultant force.



Magnitude of the resultant force

Resolving all the forces horizontally (*i.e.*, along *AB*),

$$\Sigma H = 20 \cos 0° + 30 \cos 30° + 40 \cos 60° + 50 \cos 90° + 60 \cos 120° \text{ N}$$
$$= (20 \times 1) + (30 \times 0.866) + (40 \times 0.5) + (50 \times 0) + 60 (-0.5) \text{ N}$$
$$= 36.0 \text{ N} \qquad ...(i)$$

and now resolving the all forces vertically (*i.e.*, at right angles to *AB*),

$$\Sigma V = 20 \sin 0° + 30 \sin 30° + 40 \sin 60° + 50 \sin 90° + 60 \sin 120° \text{ N}$$
$$= (20 \times 0) + (30 \times 0.5) + (40 \times 0.866) + (50 \times 1) + (60 \times 0.866) \text{ N}$$
$$= 151.6 \text{ N} \qquad ...(ii)$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(36.0)^2 + (151.6)^2} = 155.8 \text{ N} \quad \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle, which the resultant force makes with the horizontal (*i.e.*, *AB*).

We know that

$$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{151.6}{36.0} = 4.211 \quad \text{or} \quad \theta = 76.6° \quad \textbf{Ans.}$$

Note. Since both the values of ΣH and ΣV are positive, therefore actual angle of resultant force lies between 0° and 90°.

2. The following forces act at a point : (i) 20 N inclined at 30° towards North of East, (ii) 25 N towards North, (iii) 30 N towards North West, and (iv) 35 N inclined at 40° towards South of West. Find the magnitude and direction of the resultant force.

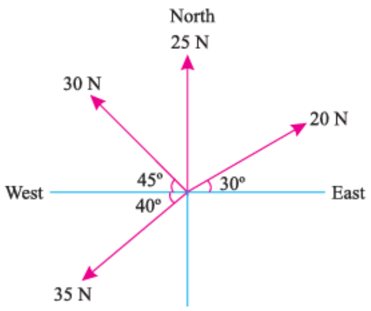


Fig. 2.6.

Magnitude of the resultant force

Resolving all the forces horizontally *i.e.*, along East-West line,

$$\Sigma H = 20 \cos 30° + 25 \cos 90° + 30 \cos 135° + 35 \cos 220° \text{ N}$$
$$= (20 \times 0.866) + (25 \times 0) + 30 (-0.707) + 35 (-0.766) \text{ N}$$
$$= -30.7 \text{ N} \qquad ...(i)$$

and now resolving all the forces vertically *i.e.*, along North-South line,

$$\Sigma V = 20 \sin 30° + 25 \sin 90° + 30 \sin 135° + 35 \sin 220° \text{ N}$$
$$= (20 \times 0.5) + (25 \times 1.0) + (30 \times 0.707) + 35 (-0.6428) \text{ N}$$
$$= 33.7 \text{ N} \qquad ...(ii)$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-30.7)^2 + (33.7)^2} = 45.6 \text{ N} \quad \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle, which the resultant force makes with the East.

We know that

$$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{33.7}{-30.7} = -1.098 \quad \text{or} \quad \theta = 47.7°$$

Since ΣH is negative and ΣV is positive, therefore resultant lies between 90° and 180°. Thus actual angle of the resultant = 180° – 47.7° = 132.3° **Ans.**

CHAPTER 5 MOMENTS & COUPLES

Course content
Concept of moment, Varignon's theorem, Principle of moments, Moment of forces about a specified axis, concept of couple - properties and effect, Moment of couple, Movement of force on rigid body, Resultant of force and couple system, Reduction of force and couple system, Parallel forces - like and unlike parallel forces, calculation of their resultant.

5.1 INTRODUCTION

In this chapter, the effects of coplanar non concurrent forces, at some other point, away from the point of intersection or their lines of action will be studied.

5.1.1 MOMENT OF A FORCE

It is the turning effect produced by a force, on the body, on which it acts. The moment of a force is equal to the product of the force and the perpendicular distance of the point, about which the moment is required and the line of action of the force.

Mathematically, Moment,

$$M = P \times d, \text{ where}$$

P= Force acting on the body, and d = Perpendicular distance between the point, about which the moment is required and the line of action of the force.

5.1.2 UNITS OF MOMENT

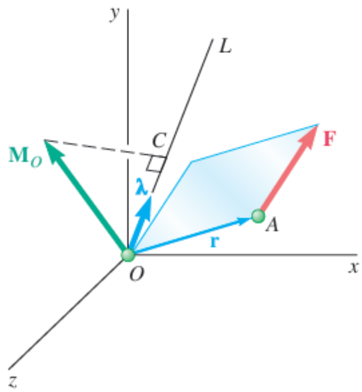
Since the moment of a force is the product of force and distance, therefore the units of the moment will depend upon the units of force and distance. Thus, if the force is in Newton and the distance is in meters, then the units of moment will be Newton-meter (briefly written as N-m). Similarly, the units of moment may be kN-m (i.e. kN × m), N-mm (i.e. N × mm) etc.

5.2 VARIGNON'S PRINCIPLE OF MOMENTS (OR LAW OF MOMENTS)

It states, If a number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point.

5.3 MOMENT OF A FORCE ABOUT AN AXIS

The moment M_{OL} of F about OL as the projection OC of the moment M_O onto the axis OL.



Moment about an axis through the origin $M_{OL} = \lambda \cdot M_O = \lambda \cdot (r \times F)$.

This shows that the moment MOL of F about the axis OL is the scalar obtained by forming the mixed triple product of λ , r, and F. We can also express MOL in the form of a determinant,

$$M_{OL} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

where λx, λy, λz = direction cosines of axis OLx, y, z = coordinates of point of application of F, Fx, Fy, Fz = components of force F The physical significance of the moment MOL of a force F about a fixed axis OL becomes more apparent if we resolve F into two rectangular components F1 and F2, with F1 parallel to OL and F2 lying in a plane P perpendicular to OL. Resolving r similarly into two components r1 and r2 and substituting for F and r, we get

$$M_{OL} = \lambda \cdot [(r_1 + r_2) \times (F_1 + F_2)]$$

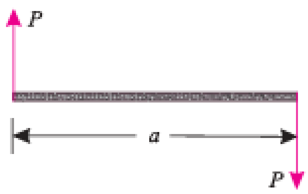
$$= \lambda \cdot (r_1 \times F_1) + \lambda \cdot (r_1 \times F_2) + \lambda \cdot (r_2 \times F_1) + \lambda \cdot (r_2 \times F_2)$$

5.4 COUPLE

A pair of two equal and unlike parallel forces (i.e. forces equal in magnitude, with lines of action parallel to each other and acting in opposite directions) is known as a couple. As a matter of fact, a couple is unable to produce any translatory motion (i.e., motion in a straight line). But it produces a motion of rotation in the body, on which it acts. The simplest example of a couple is the forces applied to the key of a lock, while locking or unlocking it.

5.4.1 ARM OF A COUPLE

The perpendicular distance (a), between the lines of action of the two equal and opposite parallel forces, is known as arm of the couple as shown in Fig.



5.4.2 MOMENT OF A COUPLE

The moment of a couple is the product of the force (i.e., one of the forces of the two equal and opposite parallel forces) and the arm of the couple. Mathematically:

$$\text{Moment of a couple} = P \times a$$

where P = Magnitude of the force, and a = Arm of the couple.

5.4.3 CHARACTERISTICS OF A COUPLE

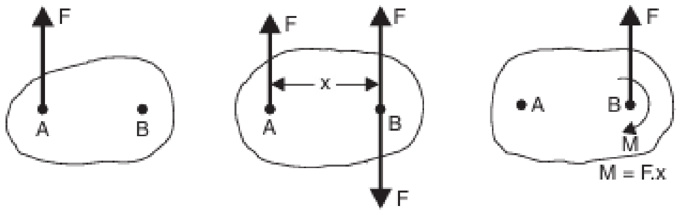
A couple (whether clockwise or anticlockwise) has the following characteristics:

1. The algebraic sum of the forces, constituting the couple, is zero.
2. The algebraic sum of the moments of the forces, constituting the couple, about any point is the same, and equal to the moment of the couple itself.
3. A couple cannot be balanced by a single force. But it can be balanced only by a couple of opposite sense.

4. Any no. of coplaner couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of the moments of all the couples.

5.5 RESOLUTION OF A FORCE INTO A FORCE AND A COUPLE

A given force F applied to a body at any point A can always be replaced by an equal and parallel force applied at another point B together with a couple which will be equivalent to the original force. This is proved as given below: Let the given force F is acting at point A as shown in Fig.

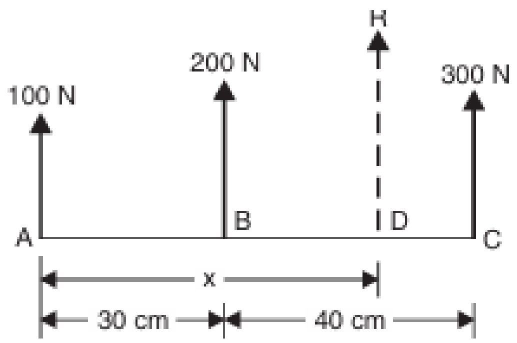


(a) (b) (c)

This force is to be replaced at the point B. Introduce two equal and opposite forces at B, each of magnitude F and acting parallel to the force at A as shown in Fig. (b). The force system of Fig. (b) is equivalent to the single force acting at A of Fig. (a). In Fig. (b) three equal forces are acting. The two forces i.e., force F at A and oppositely directed force F at B (i.e., vertically downward force at B) from a couple. The moment of this couple is F × x clockwise where x is the perpendicular distance between the lines of action of forces at A and B. The third force is acting at B in the same direction in which the force at A is acting. In Fig. (c), the couple is shown by curved arrow with symbol M. The force system of Fig. (c) is equivalent to Fig. (b). Or in other words the Fig. (c) is equivalent to Fig. (a). Hence the given force F acting at A has been replaced by an equal and parallel force applied at point B in the same direction together with a couple of moment F × x. Thus, a force acting at a point in a rigid body can be transferred to an equal and parallel force at any other point in the body, and a couple.

Sample problems

1. Three like parallel forces 100 N, 200 N and 300 N are acting at points A, B and C respectively on a straight line ABC as shown in Fig. 3.12. The distances are AB = 30 cm and BC = 40 cm. Find the resultant and also the distance of the resultant from point A on line ABC.



Force at A = 100 N B = 200 N C = 300 N Distance AB = 30 cm, BC = 40 cm. As all the forces are parallel and acting in the same direction, their result ant R is given by R = 100 + 200 + 300 = 600 N

Let the resultant is acting at a distance of x cm from the point A as shown in Fig. Now take the moments of all forces about point A.

The force 100 N is passing A, hence its moment about A will be zero.

Moment of 100 N force about A = 0

Moment of 200 N force about A = 200 × 30 = 6000 N cm (anti-clockwise) Moment of 300 N force about A = 300 × AC = 300 × 70 = 21000 N cm (anti-clockwise)

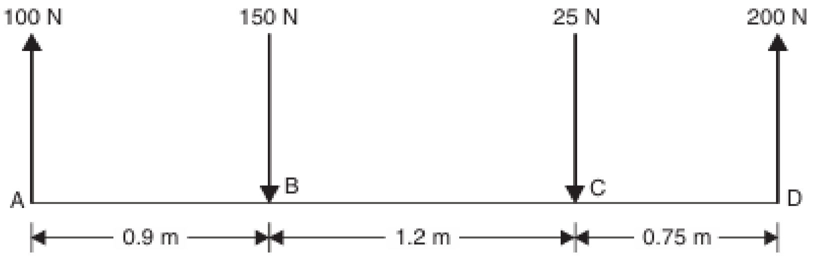
Algebraic sum of moments of all forces about A = 0 + 6000 + 21000 = 27000 N cm (anticlockwise)

Moment of resultant R about A = R × x = 600 × x N cm or

But algebraic sum of moments of all forces about A = Moment of resultant about A

27000 = 600 × x or x = 27000 = 45 cm.

2. Four parallel forces of magnitudes 100 N, 150 N, 25 N and 200 N are shown in Fig. Determine the magnitude of the resultant and also the distance of the resultant from point A.



Sol. Given :

Forces are 100 N, 150 N, 25 N and 200 N.

Distances $AB = 0.9$ m, $BC = 1.2$ m, $CD = 0.75$ m.

As all the forces are acting vertically, hence their resultant R is given by

$$R = 100 - 150 - 25 + 200$$

(Taking upward force +ve and downward as –ve)

$$= 300 - 175 = 125 \text{ N}$$

+ve sign shows that R is acting vertically upwards. To find the distance of R from point A, take the moments of all forces about point A.

Let x = Distance of R from A in metre.

As the force 100 N is passing through A, its moment about A will be zero.

Moment of 150 N force about $A = 150 \times AB$

$= 150 \times 0.9$ (clockwise) (–) $= -135$ Nm

Moment of 25 N force about $A = 25 \times AC = 25 \times (0.9 + 1.2)$

$= 25 \times 2.1$ (clockwise) (–) $= -52.5$ Nm.

Moment of 200 N force about $A = 200 \times AD$

$= 200 \times (0.9 + 1.2 + 0.75)$

$= 200 \times 2.85$ (anti-clockwise) (+) $= 570$ Nm

Algebraic sum of moments of all forces about A

$= -135 - 52.5 + 570 = 382.5$ Nm ...(*i*)

+ve sign shows that this moment is anti-clockwise. Hence the moment of resultant R about A must be 382.5 Nm, *i.e.*, moment of R should be anti-clockwise about A. The moment of R about A will be anti-clockwise if R is acting upwards and towards the right of A.

Now moment of R about $A = R \times x$. But $R = 125$

$= 125 \times x$ (anti-clockwise) (+)

$= + 125 \times x$ ...(*ii*)

Equating (*i*) and (*ii*),

$382.5 = 125 \times x$

or $x = \frac{382.5}{125}$ = **3.06 m. Ans.**

∴ Resultant (R = 125 N) will be 125 N upwards and is acting at a distance of 3.06 m to the right of point A as shown in Fig. 3.14 (*a*).

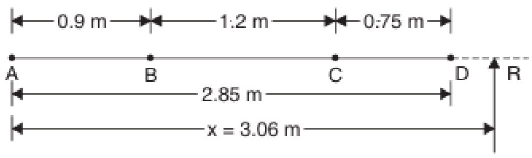


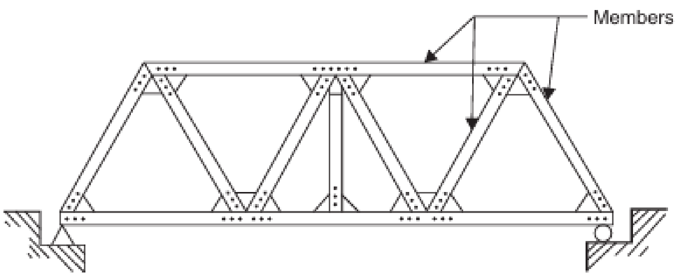
Fig. 3.14 (*a*)

CHAPTER 6 TRUSSES

Course content
Simple trusses, analysis of simple truss, Method of Joints, Method of sections

6.1 INTRODUCTION

A structure made up of several bars (or members) riveted or welded together is known as truss. The isometric view of a structure made of trusses is shown in Fig. 7.1 (a) and front view is shown in Fig. If the members of the structure are hinged or pin-joined, then the structure is known as a Frame. Hence the difference between truss and frame is that in case of truss members are riveted or welded whereas in case of frame the members are hinged or pin joined. If the frame is composed of such members which are just sufficient to keep the frame in equilibrium, when the frame is supporting an external load, then the frame is known as perfect frame. Though in actual practice the members are welded or riveted together at their joints, yet for calculation purposes the joints are assumed to be hinged or pin joined. In this chapter, we shall discuss how to determine the axial forces in the members of a perfect frame, when it is subjected to some external load.



6.2 TYPES OF FRAMES

The different types of frames are:

(i) Perfect frame, and

(ii) Imperfect frame. Imperfect frames may be a deficient frame or a redundant frame.

6.2.1 PERFECT FRAME

The frame, which is composed of such members, which are just sufficient to keep the frame in equilibrium, when the frame is supporting an external load, is known as perfect frame. for a perfect frame, the number of joints and number of members are given by, $n = 2j - 3$ where n = Number of members, and j = Number of joints.

6.2.2 IMPERFECT FRAME.

A frame in which number of members and number of joints are not given by $n = 2j - 3$ is known, as imperfect frame. This means that number of members in an imperfect frame will be either more or less than $(2j - 3)$.

(i) If the number of members in a frame are less than $(2j - 3)$, then the frame is known as deficient frame.

(ii) If the number of members in a frame are more than $(2j - 3)$, then the frame is known as redundant frame

6.3 ASSUMPTIONS AND ANALYSIS OF PLANE TRUSS

The assumptions made in finding out the axial forces in a frame are :

(i) The frame is a perfect frame

(ii) The frame carries load at the joints

(iii) All the members are pin joined.

6.3.1 ANALYSIS OF A FRAME.

Analysis of a frame consists of:

(i) Determinations of the reactions at the supports.

(ii) Determination of the axial forces in the members of the frame.

The reactions are determined by the condition that the applied load system and the induced reactions at the supports form a system in equilibrium. The forces in the members of the frame are determined by the condition that every joint should be in equilibrium and so, the forces acting at every joint should form a system in equilibrium. A frame is analysed by the following methods:

(i) Method of joints

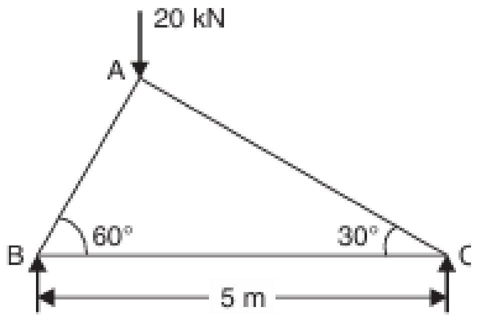
(ii) Method of sections

6.4 METHOD OF JOINTS

In this method, after determining the reactions at the supports, the equilibrium of every joint is considered. This means the sum of all the vertical forces as well as the horizontal forces acting on a joint is equated to zero. The joint should be selected in such a way that at any time there are only two members, in which the forces are unknown. The force in the member will be compressive if the member pushes the joint to which it is connected whereas the force in the member will be tensile if the member pulls the joint to which it is connected.

Sample Problem

1. Find the forces in the members AB, AC and BC of the truss shown in Fig.



Sol. First determine the reactions R_B and R_C. The line of action of load of 20 kN acting at A is vertical. This load is at a distance of $AB \times \cos 60°$ from the point B. Now let us find the distance AB.

The triangle ABC is a right-angled triangle with angle $BAC = 90°$. Hence AB will be equal to $BC \times \cos 60°$.

$\therefore \quad AB = 5 \times \cos 60° = 5 \times \frac{1}{2} = 2.5$ m

Now the distance of line of action of 20 kN from B is

$$AB \times \cos 60° \text{ or } 2.5 \times \tfrac{1}{2} = 1.25 \text{ m.}$$

Taking the moments about B, we get

$$R_C \times 5 = 20 \times 1.25 = 25$$

$\therefore \quad R_C = \dfrac{25}{5} = 5$ kN

and $\quad R_B = \text{Total load} - R_C = 20 - 5 = 15$ kN

Now let us consider the equilibrium of the various joints.

Joint B

Let F_1 = Force in member AB

F_2 = Force in member BC

Let the force F_1 is acting towards the joint B and the force F_2 is acting away* from the joint B as shown in Fig. 7.6. (The reaction R_B is acting vertically up. The force F_2 is horizontal. The reaction R_B will be balanced by the vertical component of F_1. The vertical component of F_1 must act downwards to balance R_B. Hence F_1 must act towards the joint B so that its vertical component is downward. Now the horizontal component of F_1 is towards the joint B. Hence force F_2 must act away from the joint to balance the horizontal component of F_1).

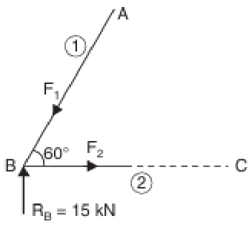


Fig. 7.6

Resolving the forces acting on the joint B, vertically

$$F_1 \sin 60° = 15$$

$$\therefore \quad F_1 = \frac{15}{\sin 60°} = \frac{15}{0.866} = 17.32 \text{ kN (Compressive)}$$

As F_1 is pushing the joint B, hence this force will be compressive. Now resolving the forces horizontally, we get

$$F_2 = F_1 \cos 60° = 17.32 \times \tfrac{1}{2} = 8.66 \text{ kN (tensile)}$$

As F_2 is pulling the joint B, hence this force will be tensile.

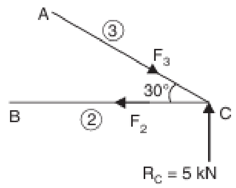


Fig. 7.7

Joint C

Let F_3 = Force in the member AC

F_2 = Force in the member BC

The force F_2 has already been calculated in magnitude and direction. We have seen that force F_2 is tensile and hence it will pull the joint C. Hence it must act away from the joint C as shown in Fig. 7.7.

Resolving forces vertically, we get

$$F_3 \sin 30° = 5 \text{ kN}$$

$$\therefore \quad F_3 = \frac{5}{\sin 30°} = 10 \text{ kN (Compressive)}$$

As the force F_3 is pushing the joint C, hence it will be compressive.

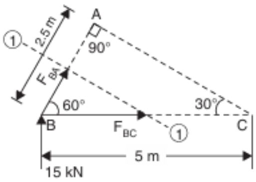
6.5 METHOD OF SECTION

When the forces in a few members of a truss are to be determined, then the method of section is mostly used. This method is very quick as it does not involve the solution of other joints of the truss. In this method, a section line is passed through the members, in which forces are to be determined as shown in Fig. 7.25. The section line should be drawn in such a way that it does not cut more than three members in which the forces are unknown. The part of the truss, on any one side of the section line, is treated as a free body in equilibrium under the action of external forces on that part and forces in the members cut by the section line. The unknown forces in the members are then determined by using equations of equilibrium as

$$\Sigma Fx = 0, \Sigma Fy = 0 \text{ and } \Sigma M = 0.$$

Sample Problem

1. Find the forces in the members AB and AC of the truss shown in Fig. using method of section.



Sol. First determine the reaction R_B and R_C.

The distance of line of action of 20 kN from point B is

$AB \times \cos 60°$ or $2.5 \times \frac{1}{2} = 1.25$ m

Taking moments about point B, we get

$$R_C \times 5 = 20 \times 1.25$$

$$\therefore \quad R_C = \frac{20 \times 1.25}{5} = 5 \text{ kN}$$

and $$R_B = 20 - 5 = 15 \text{ kN}$$

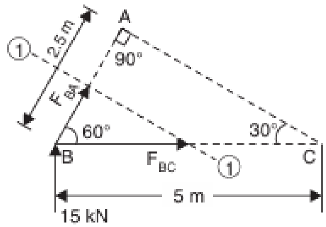


Fig. 7.26

Now draw a section line (1-1), cutting the members AB and BC in which forces are to be determined. Now consider the equilibrium of the left part of the truss. This part is shown in Fig. 7.27.

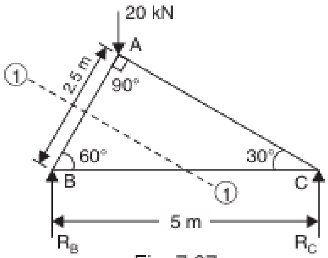


Fig. 7.27

Let the directions of F_{BA} and F_{BC} are assumed as shown in Fig. 7.27.

Now taking the moments of all the forces acting on the left part about point C, we get

$$15 \times 5 + (F_{BA} \times AC)^* = 0$$

($\because$ The perpendicular distance between the line of action of F_{BA} and point C is equal to AC)

or $$75 + F_{BA} \times 5 \times \cos 30° = 0 \quad (\because \; AC = BC \times \cos 30°)$$

or $$F_{BA} = \frac{-75}{5 \times \cos 30°} = -17.32 \text{ kN}$$

The negative sign shows that F_{BA} is acting in the opposite direction (*i.e.*, towards point B). Hence force F_{BA} will be a compressive force.

$\therefore$ F_{BA} = **17.32 kN** (Compressive). **Ans.**

Again taking the moments of all the forces acting on the left part about point A, we get

15 × Perpendicular distance between the line of action of 15 kN and point C

= F_{BC} × Perpendicular distance between F_{BC} and point A

$$15 \times 2.5 \times \cos 60° = F_{BC} \times 2.5 \times \sin 60°$$

$$\therefore \quad F_{BC} = \frac{15 \times 2.5 \times \cos 60°}{2.5 \times \sin 60°} = \frac{15 \times 0.5}{0.866}$$

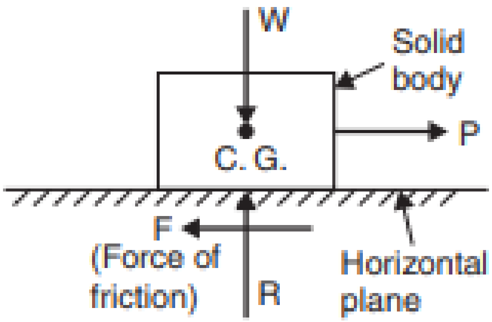
= **8.66 kN** (Tensile). **Ans.**

CHAPTER 7 FRICTION

Course content
Concept of friction, Characteristics of Dry friction, Laws of Coulomb friction, limiting friction, coefficient of friction; sliding friction and rolling friction, Belt friction, Ladder friction.

7.1 INTRODUCTION

When a solid body slides over a stationary solid body, a force is exerted at the surface of contact by the stationary body on the moving body. This force is called the force of friction and is always acting in the direction opposite to the direction of motion. The property of the bodies by virtue of which a force is exerted by a stationary body on the moving body to resist the motion of the moving body is called friction. Friction acts parallel to the surface of contact and depends upon the nature of surface of contact.



Let W = Weight of body acting through C.G. downward, R = Normal reaction of body acting through C.G. upward, P = Force acting on the body through C.G. and parallel to the horizontal surface. If P is small, the body will not move as the force of friction acting on the body in the direction opposite to P will be more than P. But if the magnitude of P goes on increasing, a stage comes when the solid body is on the point of motion. At this stage, the force of friction acting on the body is called limiting force of friction. The limiting force of friction is denoted by F. Resolving the forces on the body vertically and horizontally, we get

$$R = W$$

$$F = P$$

If the magnitude of P is further increased the body will start moving. The force of friction, acting on the body when the body is moving, is called kinetic friction.

7.2 CO-EFFICIENT OF FRICTION (M)

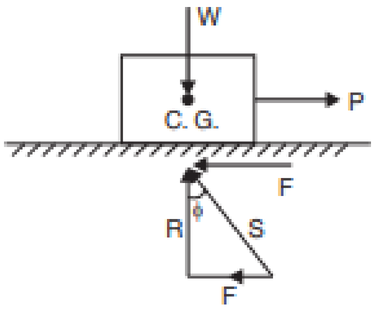
It is defined as the ratio of the limiting force of friction (F) to the normal reaction (R) between two bodies. It is denoted by the symbol μ. Thus

$$\mu = \frac{\text{Limiting force of friction}}{\text{Normal reaction}} = \frac{F}{R}$$

$$F = \mu R$$

7.3 ANGLE OF FRICTION (Φ)

It is defined as the angle made by the resultant of the normal reaction (R) and the limiting force of friction (F) with the normal reaction (R). It is denoted by φ. Fig. 6.2 shows a solid body resting on a rough horizontal plane.



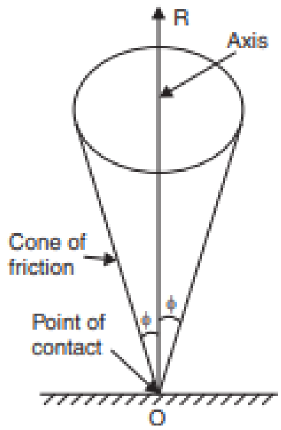
Let S = Resultant of the normal reaction (R) and limiting force of friction (F) Then angle of friction = φ = Angle between S and R

$$\tan \phi = \frac{F}{R} = \frac{\mu R}{R}$$

= μ = Co-efficient of friction, Thus the tangent of the angle of friction is equal to the co-efficient of friction.

7.4 CONE OF FRICTION

It is defined as the right circular cone with vertex at the point of contact of the two bodies (or surfaces), axis in the direction of normal reaction (R) and semi-vertical angle equal to angle of friction (φ). Fig. shows the cone of friction in which, O = Point of contact between two bodies R = Normal reaction and also axis of the cone of friction φ = Angle of friction.



7.5 TYPES OF FRICTION

The friction is divided into following two types depending upon the nature of the two surfaces in contact:

1. Static and dynamic friction

2. Wet and dry friction.

7.5.1 STATIC AND DYNAMIC FRICTION

If the two surfaces, which are in contact, are at rest, the force experienced by one surface is called static friction. But if one surface starts moving and the other is at rest, the force experienced by the moving surface is called dynamic friction.

7.5.2 WET AND DRY FRICTION

If between two surfaces, which are in contact, lubrication (oil or grease) is used, the friction that exists between two surfaces is known wet friction. But if no lubrication (oil or grease) is used, then the friction between two surfaces is called Dry Friction or Solid Friction.

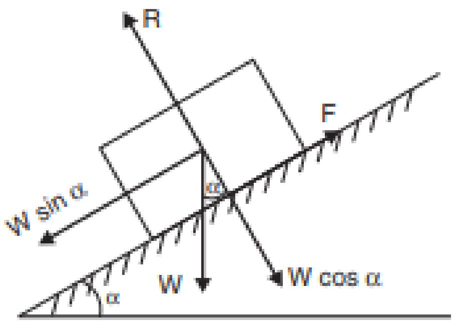
7.6 COULOMB'S LAWS OF FRICTION

The friction that exists between two surfaces which are not lubricated, is known as solid friction. The two surfaces may be at rest or one of the surface is moving and the other surface is at rest. The following are the laws of solid friction:

1. The force of friction acts in the opposite direction in which surface is having tendency to move.
2. The force of friction is equal to the force applied to the surface, so long as the surface is at rest.
3. When the surface is on the point of motion, the force of friction is maximum, and this maximum frictional force is called the limiting friction force.
4. The limiting frictional force bears a constant ratio to the normal reaction between two surfaces.
5. The limiting frictional force does not depend upon the shape and areas of the surfaces in contact.
6. The ratio between limiting friction and normal reaction is slightly less when the two surfaces are in motion.
7. The force of friction is independent of the velocity of sliding.

7.7 ANGLE OF REPOSE

The angle of repose is defined as the maximum inclination of a plane at which a body remains in equilibrium over the inclined plane by the assistance of friction only.



Let R = Normal reaction acting at right angle to the inclined plane.

α = Inclination of the plane with the horizontal F = Frictional force acting upward along the plane.

Let the angle of inclination (α) be gradually increased, till the body just starts sliding down the plane.

This angle of inclined plane, at which a body just begins to slide down the plane, is called angle of repose. Resolving the forces along the plane,

$$W \sin \alpha = F \ldots(i)$$

Resolving the forces normal to the plane,

$$W \cos \alpha = R \ldots(ii)$$

Dividing equation (i) by equation (ii),

$$\frac{W \sin \alpha}{W \cos \alpha} = \frac{F}{R} \quad \text{or} \quad \tan \alpha = \frac{F}{R}$$

But, we know

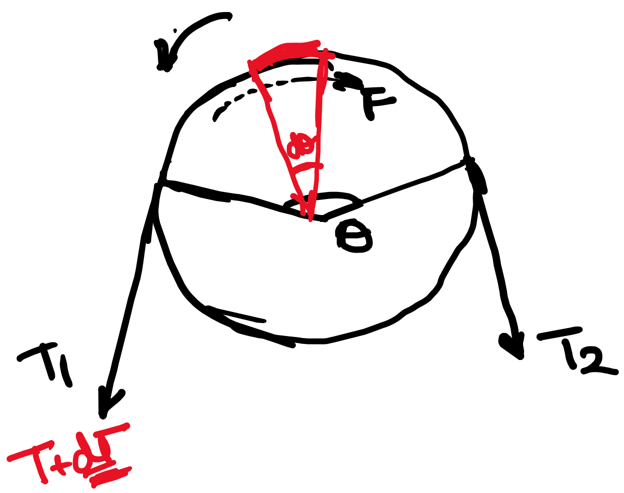
$$\tan \phi = \frac{F}{R}$$

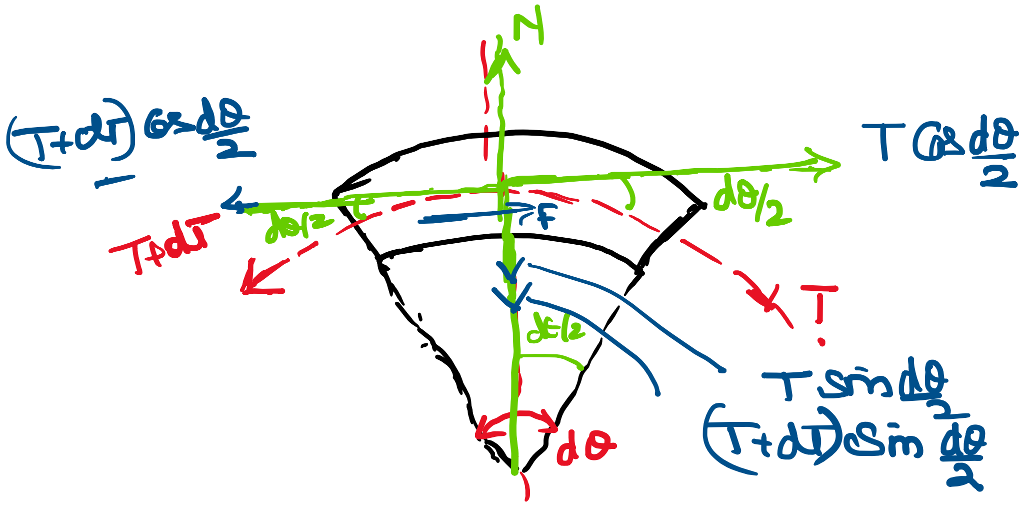
Hence from equations

tan α = tan φ or α = φ or Angle of repose = Angle of friction.

7.8 BELT FRICTION

A belt is passing over a pulley and hence the belt is in contact with the surface of the pulley. If the surface of the pulley is perfectly smooth, the tension in the belt on both sides* of the pulley will be same (i.e. the tension throughout the belt will be constant). Also for the perfectly smooth surface, there will be no frictional resistance and hence no driving torque** will be developed. But if the surface of the pulley is rough, the tension in the belt will not be constant. The tension will vary throughout the length of the belt which is in contact with pulley. This variation in tension is due to frictional resistance. The frictional resistance depends on the co-efficient of friction (i.e. value of μ) between the belt and pulley surface.





Resolving all the forces acting on the belt MN in the horizontal direction,

$$R = T \sin \delta\theta \; 2 + (T + \delta T) \sin \delta\theta$$

Since the angle δθ is very small, sin δθ/2 can be written as δθ/2 . Hence the above equation becomes as

$$R = T \times \delta\theta/2 + (T + \delta T) \times \delta\theta/2 = T \times \delta\theta/2 + T \times \delta\theta/2 + \delta T \times \delta\theta/2$$

$$R = T \times \delta\theta$$

Now resolving all the forces vertically,

$$F = (T + \delta T) \cos \delta\theta/2 - T \cos \delta\theta/2$$

Since δθ is very small, hence cos δθ/2 reduces to unity i.e., 1. Hence the above equation becomes as

$$F = (T + \delta T) - T = \delta T \text{ or}$$

$$\mu R = \delta T$$

$$\text{or } R = \delta \; \mu \; T \; \ldots\text{(ii)}$$

Equating the two values of R given by equations (i) and (ii),

$$T \times \delta\theta = \delta\mu/T$$

$$\text{or } \delta T \; T = \mu \; . \; \delta\theta$$

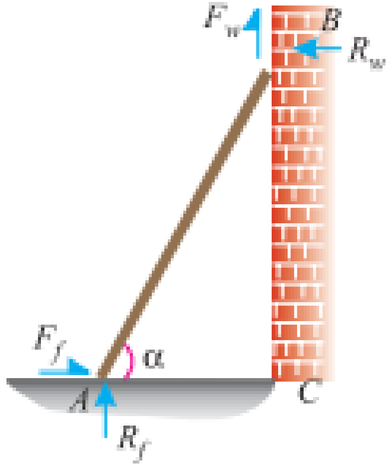
Integrating the above equation between the limits T_2 and T_1,

$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \int \mu \, . \, d\theta = \mu \int d\theta$$

$$\log_e \frac{T_1}{T_2} = \mu \times \theta$$

$$\frac{T_1}{T_2} = e^{\mu \times \theta}$$

7.9 LADDER FRICTION



The ladder is a device for climbing or scaling on the roofs or walls. It consists of two long uprights of wood, iron or rope connected by a number of cross pieces called rungs.

Consider a ladder AB resting on the rough ground and leaning against a wall.

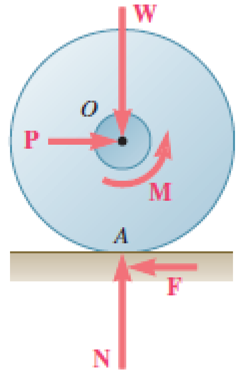
As the upper end of the ladder tends to slip downwards, therefore the direction of the force of friction between the ladder and the wall (F_w) will be upwards as shown in the figure. Similarly, as the lower end of the ladder tends to slip away from the wall, the direction of the force of friction between the ladder and the floor (F_f) will be towards the wall as shown in the figure. Since the system is in equilibrium, therefore the algebraic sum of the horizontal and vertical components of the forces must also be equal to zero.

7.10ROLLING RESISTANCE

The wheel is one of the most important inventions of our civilization. Among many other uses, with a wheel we can move heavy loads with relatively little effort. Because the point where the wheel is in contact with the ground at any given instant has no relative motion with respect to the ground, use of the wheel avoids the large friction forces that would arise if the load were in direct contact with the ground. However, some resistance to the wheel' s motion does occur.

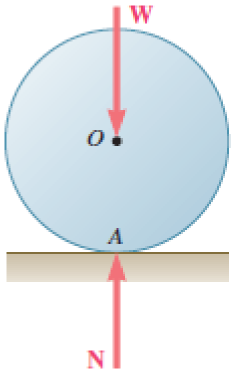
This resistance has two distinct causes. It is due to (1) a combined effect of axle friction and friction at the rim and (2) the fact that the wheel and the ground deform, causing contact between the wheel and ground to take place over an area rather than at a single point.

To understand better the first cause of resistance to the motion of a wheel, consider a railroad car supported by eight wheels mounted on axles and bearings. Assume that the car is moving to the right at constant speed along a straight horizontal track. The free-body diagram of one of the wheels is shown in Fig. *a*.



(a) Effect of axle friction

The forces acting on the free body include the load **W** supported by the wheel and the normal reaction **N** of the track. Because **W** passes through the center *O* of the axle, we represent the frictional resistance of the bearing by a counterclockwise couple **M**. Then, to keep the free body in equilibrium, we must add two equal and opposite forces **P** and **F**, forming a clockwise couple of moment −**M**. Force **F** is the friction force exerted by the track on the wheel, and **P** represents the force that should be applied to the wheel to keep it rolling at constant speed. Note that the forces **P** and **F** would not exist if there were no friction between the wheel and the track. The couple **M** representing the axle friction would then be zero; the wheel would slide on the track without turning in its bearing. The couple **M** and the forces **P** and **F** also reduce to zero when there is no axle friction. For example, a wheel that is not held in bearings but rolls freely and at constant speed on horizontal ground (Fig. *b*) is subjected to only two forces: its own weight **W** and the normal reaction **N** of the ground. No friction force acts on the wheel regardless of the value of the coefficient of friction between the wheel and ground. Thus, a wheel rolling freely on horizontal ground should keep rolling indefinitely.

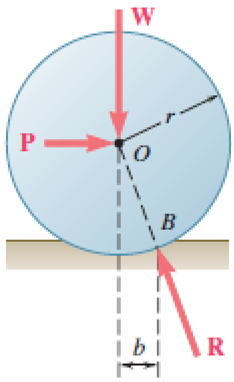


(b) Free wheel

Experience, however, indicates that a free wheel does slow down and eventually comes to rest. This is due to the second type of resistance mentioned at the beginning of this section, known as **rolling resistance**. Under load **W**, both the wheel and the ground deform slightly, causing the contact between wheel and ground to take place over a certain area. Experimental evidence shows that the resultant of the forces exerted by the ground on the wheel over this area is a force **R** applied at a point *B,* which is not located directly under the center *O* of the wheel but slightly in front of it (Fig. *c*). To balance the moment of **W** about *B* and to keep the wheel rolling at constant speed, it is necessary to apply a horizontal force **P** at the center of the wheel. Setting $\Sigma M_B = 0$,

$$Pr = Wb$$

where r = radius of wheel b = horizontal distance between *O* and *B.* The distance *b* is commonly called the **coefficient of rolling resistance**.



(*c*) Rolling resistance

Note that *b* is not a dimensionless coefficient, because it represents a length; *b* is usually expressed in inches or in millimeters. The value of *b* depends upon several parameters in a manner that has not yet been clearly established. Values of the coefficient of rolling resistance vary from about 0.01 in. or 0.25 mm for a steel wheel on a steel rail to 5.0 in. or 125 mm for the same wheel on soft ground.

Sample Problems

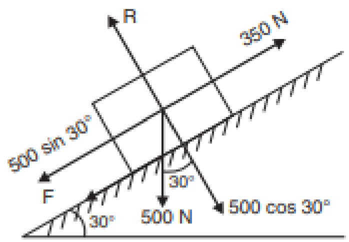
1. A body of weight 500 N is pulled up an inclined plane, by a force of 350 N. The inclination of the plane is 30° to the horizontal and the force is applied parallel to the plane. Determine the co-efficient of friction.

Given:

Weight of body,

W = 500 N

Force applied, P = 350 N Inclination, α = 30°



Let μ = Co-efficient of Q friction

R = Normal reaction

F = Force of friction = μR

The body is in equilibrium under the action of the forces shown in Fig.

Resolving the forces along the plane,

500 sin 30° + F = 350 or

500 sin 30° + μR = 350 ...(i)

Resolving forces normal to the plane,

R = 500 cos 30° = 500 × .866 = 433 N

Substituting the value of R in equation (i),

500 sin 30° + μ × 433 = 350 or 500 × 0.5 + 433

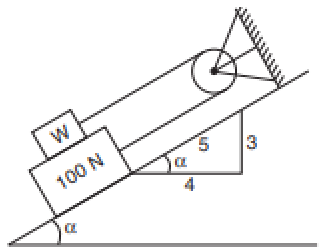
μ = 350 or 433

μ = 350 – 500 × 0.5 = 350 – 250 = 100

∴ μ = 100/433 = 0.23.

2. In the Fig., the co-efficient of friction is 0.2 between the rope and fixed pulley, and between other surfaces of contact, μ = 0.3. Determine the minimum weight W to prevent the downward motion of the 100 N body.

Given :



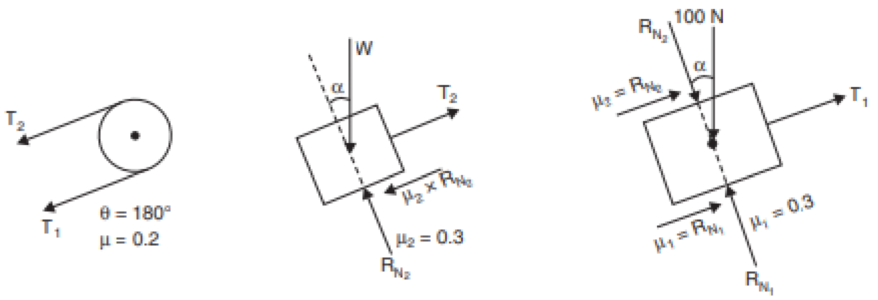
Co-efficient of friction between rope and pulley, μ = 0.2

tan α = 3/4

∴ cos α = 4/5 and sin α = 3/5

Co-efficient of friction between other surfaces, $\mu_1 = \mu_2 = 0.3$.

first draw the Free-Body Diagrams (FBD) of pulley, weight W and body of weight 100 N.



(i) FBD of pulley (ii) FBD of W (iii) FBD of body of weight 100 N

(i) Consider the FBD of pulley

Angle of contact, $\theta = 180° = \pi$ radian

Using equation, $T_1 / T_2 = e^{\mu \times \theta}$

$\therefore T_1 = 1.874 T_2$ ...(i)

(As the body of weight 100 N will have tendency of moving downwards, hence T_1 will be more than T_2).

(ii) Now consider the FBD of weight W

The forces acting on weight W are shown in Fig.(b). They are :

(a) Tension T_2

(b) Weight W

(c) Normal reaction on lower surface of contact, R_{N2} (d) Force of friction,

$\mu \times R_{N2} = 0.3 \times R_{N2}$

For equilibrium, the resultant force along the inclined plane and normal to the inclined plane should be zero.

$$\Sigma(\text{Forces normal to plane}) = 0$$

$$\therefore W\cos\alpha - R_{N2} = 0$$

$$\therefore R_{N2} = W\cos\alpha = W \times 4/5$$

Σ(Forces along the plane) = 0

$\therefore T_2 - \mu_2 \times R_{N2} - W\sin\alpha = 0 \therefore T_2 = \mu_2 \times R_{N2} + W\sin\alpha = 0.3 \times R_{N2} + W \times 3/5$

$= 0.3 \times 4/5\ W + 3/5\ W$

$= 0.24W + 0.6W = 0.84W$

Now consider the FBD of body of weight 100 N. The forces acting on the body of weight 100 N are shown in Fig.(iii).

For equilibrium, Σ(Forces normal to plane) = 0

$R_{N1} - R_{N2} - 100 \cos \alpha = 0$

$= 4/5\ W + 100 \times 4/5$

Σ(Forces along the plane) = 0

$T_1 + \mu_1 \times R_{N1} + \mu_2 \times R_{N2} - 100 \sin \alpha = 0$

$\therefore T_1 = 100 \sin \alpha - \mu_1 \times R_{N1} - \mu_2 \times R_N$

$= 100 \times 3/5 - 0.3 \times (0.8W + 80) - 0.3 \times (0.8W)$

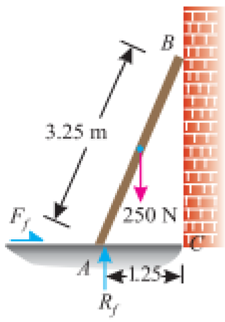
Substituting the values of T1 and T2 in equation

W = 36/2.054 = 17.52 N.

4. A uniform ladder of length 3.25 m and weighing 250 N is placed against a smooth vertical wall with its lower end 1.25 m from the wall. The coefficient of friction between the ladder and floor is 0.3. What is the frictional force acting on the ladder at the point of contact between the ladder and the floor? Show that the ladder will remain in equilibrium in this position.

Given: Length of the ladder (l) = 3.25 m; Weight of the ladder (w) = 250 N; Distance between the lower end of ladder and wall = 1.25 m and coefficient of friction between the ladder and floor (μf) = 0.3.

Frictional force acting on the ladder. The forces acting on the ladder are shown in Fig.



let F_f = Frictional force acting on the ladder at the Point of contact between the ladder and floor, and R_f = Normal reaction at the floor.

Since the ladder is placed against a smooth vertical wall, therefore there will be no friction at the point of contact between the ladder and wall. Resolving the forces vertically,

$R_f = 250$ N

From the geometry of the figure,

BC = sqrt $\{(3.25)^2 - (1.25)^2\}$=3.0 m

Taking moments about B and equating the same,

$F_f \times 3 = (R_f \times 1.25) - (250 \times 0.625) = (250 \times 1.25) - 156.3 = 156.2$ N

$\therefore F_f = 156.2/3 = 52.1$ N

Equilibrium of the ladder We know that the maximum force of friction available at the point of contact between the ladder and the floor = μRf = 0.3 × 250 = 75 N Thus we see that the amount of the force of friction available at the point of contact (75 N) is more than the force of friction required for equilibrium (52.1 N). Therefore, the ladder will remain in an equilibrium position.

CHAPTER 8 CENTRE OF GRAVITY AND MOMENT OF INERTIA

Course content
Concept of gravity, gravitational force, centroid and centre of gravity, centroid for regular lamina and centre of gravity for regular solids. Position of centre of gravity of compound bodies and centroid of composite area. CG of bodies with portions removed, Moment of Inertia: First and second moment of area; Radius of gyration, Moment of inertia of simple and composite bodies.

8.1 INTRODUCTION

It has been established, since long, that every particle of a body is attracted by the earth towards its centre. The force of attraction, which is proportional to the mass of the particle, acts vertically downwards and is known as weight of the body. As the *distance between the different particles of a body and the centre of the earth is the same, therefore these forces may be taken to act along parallel lines. This point through which the whole weight of the body acts, irrespect of its position, is known as centre of gravity (briefly written as C.G.). It may be noted that every body has one and only one centre of gravity.

8.2 CENTROID

The point at which the total area of a plane figure (like rectangle, square, triangle, quadrilateral, circle etc.) is assumed to be concentrated, is known as the centroid of that area. The centroid is also represented by C.G. or simply G. The centroid and centre of gravity are at the same point.

8.3 CENTRE OF MASS

The point at which the total mass of a body is assumed to be concentrated. A body is having only one centre of mass for all positions of the body.

8.4 CENTROID OR CENTRE OF GRAVITY OF SIMPLE PLANE FIGURES

(i) The centre of gravity (C.G.) of a uniform rod lies at its middle point.

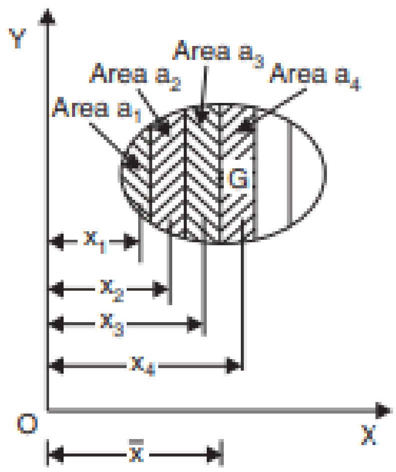
(ii) The centre of gravity of a triangle lies at the point where the three medians* of the triangle meet.

(iii) The centre of gravity of a rectangle or of a parallelogram is at the point, where its diagonal meet each other. It is also the point of intersection of the lines joining the middle points of the opposite sides.

(iv) The centre of gravity of a circle is at its centre.

8.5 CENTROID (OR CENTRE OF GRAVITY) OF AREAS OF PLANE FIGURES BY THE METHOD OF MOMENTS

Fig. shows a plane figure of total area A whose centre of gravity is to be determined.



Let this area A is composed of a number of small areas a1, a2, a3, a4, ... etc.

∴ A = a1 + a2 + a3 + a4 + ...

Let x1 = The distance of the C.G. of the area a1 from axis OY

x2 = The distance of the C.G. of the area a2 from axis OY

x3 = The distance of the C.G. of the area a3 from axis OY

x4 = The distance of the C.G. of the area a4 from axis OY and so on.

The moments of all small areas about the axis OY

= a1x1 + a2x2 + a3x3 + a4x4 + ...

Let G is the centre of gravity of the total area A whose distance from the axis OY is $\bar{x}$.

Then moment of total area about OY = A $\bar{x}$ ...(ii)

The moments of all small areas about the axis OY must be equal to the moment of total area about the same axis.

$a1x1 + a2x2 + a3x3 + a4x4 + ... = A\bar{x}$

$\bar{x}= (a1x1 + a2x2 + a3x3 + a4x4 + ...)/A$

where A = a1 + a2 + a3 + a4

the moments of the small areas about the axis OX and also the moment of total area about the axis OX,

$\bar{y}= (a1y1 + a2y2 + a3y3 + a4y4 + ...)/A$

Centre of Gravity of Areas of Plane Figures by Integration Method

$$\bar{x} = \frac{\sum a_i x_i}{\sum a_i} \quad \text{and} \quad \bar{y} = \frac{\sum a_i y_i}{\sum a_i}$$

$$\bar{x} = \frac{\int x * dA}{\int dA}$$

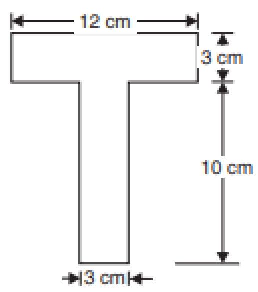
$$\bar{y} = \frac{\int y * dA}{\int dA}$$

8.6 CENTROID OF COMPOSITE BODIES

The centre of gravity of composite bodies or sections like T-section, I-section, L-sections etc. are obtained by splitting them into rectangular components.

Example: Find the centre of gravity of the T-section shown in Fig.

The given T-section is split up into two rectangles ABCD and EFGH as shown in Fig. The given T-section is symmetrical about Y-Y axis. Hence the C.G. of the section will lie on this axis. The lowest line of the figure is line GF. Hence the moments of the areas are taken about this line GF, which is the axis of reference in this case.



Let y = The distance of the C.G. of the T-section from the bottom line GF (which is axis of reference)

a1 = Area of rectangle ABCD = 12 × 3 = 36 cm^2

y1 = Distance of C.G. of area a1 from bottom line GF = 10 + 3/2 = 11.5 cm

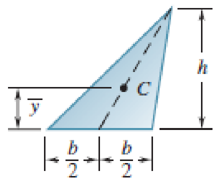
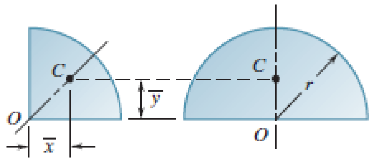
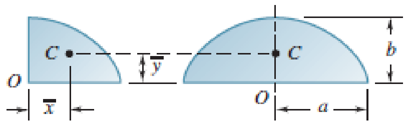
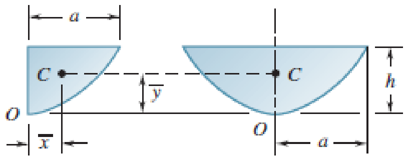
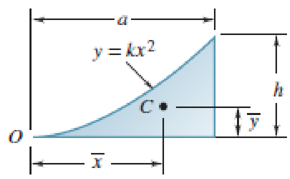
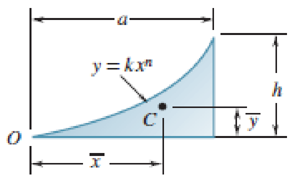
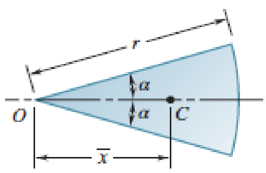
a2 = Area of rectangle EFGH = 10 × 3 = 30 cm²

y2 = Distance of C.G. of area a2 from bottom line GF = 10/2 = 5 cm.

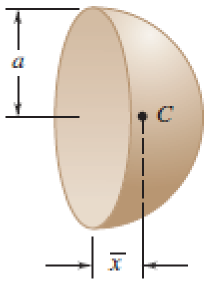
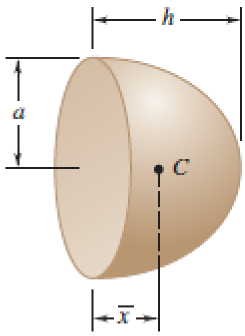
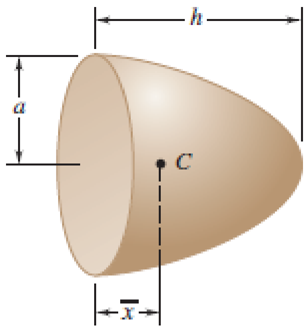
using equation, $\bar{y}$= (a1y1 + a2y2 + a3y3 + a4y4 + ...)/A= 8.545 cm

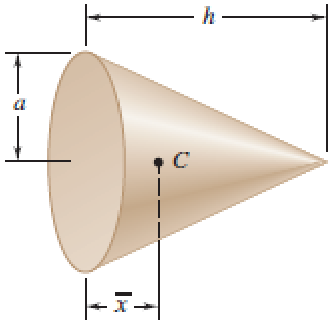
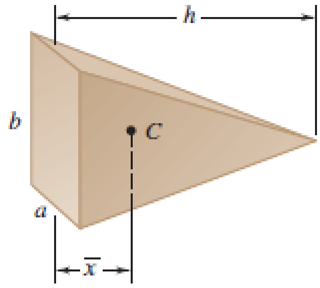
8.6.1 CENTROID OF COMMON PLANE LAMINAS

Flowing list shows the centroid of commonly encountered plane laminas.

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

8.6.2 CENTER OF GRAVITY OF REGULAR SOLIDS

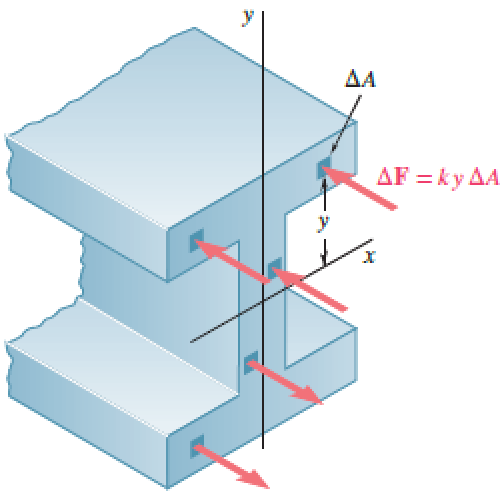
Shape		$\bar{x}$	Volume
Hemisphere		$\frac{3a}{8}$	$\frac{2}{3}\pi a^3$
Semiellipsoid of revolution		$\frac{3h}{8}$	$\frac{2}{3}\pi a^2 h$
Paraboloid of revolution		$\frac{h}{3}$	$\frac{1}{2}\pi a^2 h$

Cone		$\frac{h}{4}$	$\frac{1}{3}\pi a^2 h$
Pyramid		$\frac{h}{4}$	$\frac{1}{3}abh$

8.7 MOMENT OF INERTIA

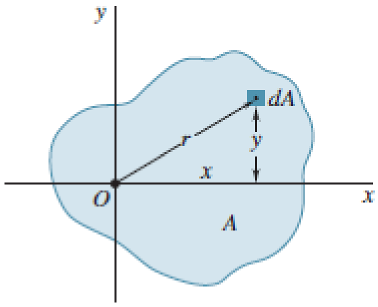
The product of the area (or mass) and the square of the distance of the centre of gravity of the area (or mass) from an axis is known as moment of inertia of the area (or mass) about that axis. Moment of inertia is represented by I.

$$I_x = \int y^2 dA \quad I_y = \int x^2 dA$$



8.8 POLAR MOMENT OF INERTIA

The product of the area (or mass) and the square of the distance of the centre of gravity of the area (or mass) from an axis perpendicular to the plane of the area is known as polar moment of inertia and is represented by J.



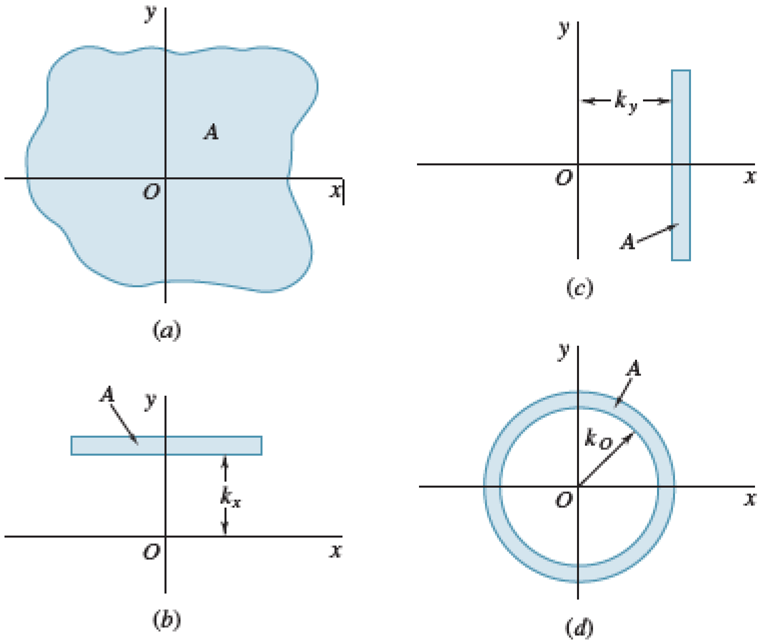
$$J_O = \int r^2 dA$$

8.9 RADIUS OF GYRATION OF AN AREA

Consider an area A that has a moment of inertia I_x with respect to the x axis. Imagine that we concentrate this area into a thin strip parallel to the x axis. If the concentrated area A is to have the same moment of inertia with respect to the x axis, the strip should be placed at a distance k_x from the x axis, where k_x is defined by the relation.

$$I_x = k_x^2 A$$

$$k_x = \sqrt{\frac{I_x}{A}}$$



8.10 THE PARALLEL-AXIS THEOREM

The moment of inertia I of an area with respect to any given axis AA' is equal to the moment of inertia $\bar{I}$ of the area with respect to a centroidal axis BB parallel to AA' *plus* the product of the area A and the square of the distance d between the two axes. This theorem is known as the **parallel-axis theorem**.

$$I = \bar{I} + Ad^2$$

8.11 PERPENDICULAR AXIS THEOREM

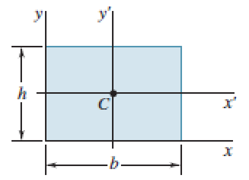
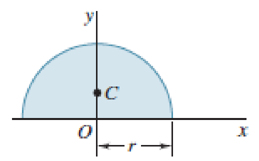
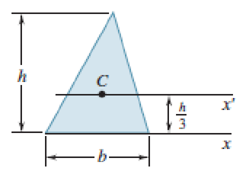
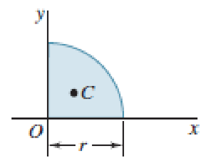
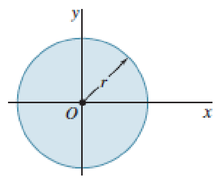
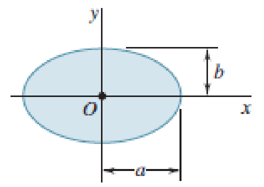
Theorem of the perpendicular axis states that if IXX and IYY be the moment of inertia of a plane section about two mutually perpendicular axis X-X and Y-Y in the plane of the section, then the moment of inertia of the section IZZ about the axis Z-Z, perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by

$$I_{ZZ} = I_{XX} + I_{YY}.$$

The moment of inertia I_{ZZ} is also known as polar moment of inertia.

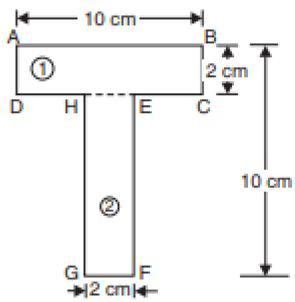
8.12 MOMENT OF INERTIA OF SIMPLE PLANE LAMINAS

Following are the moment of inertias of various plane laminas.

Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$	Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$	Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$	Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$

Sample Problems

1. A T-section of dimensions 10 × 10 × 2 cm. Determine the moment of inertia of the section about the horizontal and vertical axes, passing through the centre of gravity of the section. Also find the polar moment of inertia of the given T-section.

   Sol. The given section is symmetrical about the axis Y-Y and hence the C.G. of the section will lie on Y-Y axis. The given section is split up into two rectangles ABCD and EFGH for calculating the C.G. of the section.

Let y = Distance of the C.G. of the section from the bottom line GF

a1 = Area of rectangle ABCD = 10 × 2 = 20 cm^2

y2 = Distance of C.G. of the area a1 from the bottom line GF = 9 cm

a2 = Area of rectangle EFGH = 8 × 2 = 16 cm^2

y2 = Distance of C.G. of rectangle EFGH from the bottom line GF = 8/2 = 4 cm

therefore, $$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{20 \times 9 + 16 \times 4}{20 + 16} = \frac{180 + 64}{36} = \frac{244}{36} = 6.777 \text{ cm.}$$

Hence the C.G. of the given section lies at a distance of 6.777 cm from GF.

Let I_{G1} = Moment of inertia of rectangle (1) about the horizontal axis and passing through its C.G.

I_{G2} = Moment of inertia of rectangle (2) about the horizontal axis and passing through the C.G. of the rectangle (2)

h_1 = The distance between the C.G. of the given section and the C.G. of the rectangle (1) = $y_1 - y$ = 9.0 – 6.777 = 2.223 cm

h_2 = The distance between the C.G. of the given section and the C.G. of the rectangle (2) = $y - y_2$ = 6.777 – 4.0 = 2.777 cm.

$$I_{G_1} = \frac{10 \times 2^3}{2} = 6.667 \text{ cm}^4$$

$$I_{G_2} = \frac{2 \times 8^3}{12} = 85.333 \text{ cm}^4.$$

From the theorem of parallel axes, the moment of inertia of the rectangle (1) about the horizontal axis passing through the C.G. of the given section

$I_{G1} + a_1 h_1^2 = 6.667 + 20 \times (2.223)^2 = 6.667 + 98.834 = 105.501$ cm4.

Similarly, the moment of inertia of the rectangle (2) about the horizontal axis passing through the C.G. of the given section

$I_{G2} + a_2 h_2^2 = 85.333 + 16 \times (2.777)^2 = 85.333 + 123.387 = 208.72$ cm4.

∴ The moment of inertia of the given section about the horizontal axis passing through the C.G. of the given section is,

$I_{xx} = 105.501 + 208.72 = 314.221$ cm4.

The moment of inertia of the given section about the vertical axis passing through the C.G. of the given section is

$$I_{yy} = \frac{2 \times 10^3}{12} + \frac{8 \times 2^3}{12}$$
$$= 166.67 + 5.33 = \mathbf{172\ cm^4.}$$

the polar moment of inertia (Izz)

$I_{zz} = I_{xx} + I_{yy} = 314.221 + 221 = 486.221$ cm4.

CHAPTER 9 SIMPLE LIFTING MACHINES

Course content
Concept of machine, mechanical advantage, velocity ratio and efficiency of a machine, their relationship, law of machine, Simple machines: Wheel and axle, pulley systems, Simple screw jacks

9.1 INTRODUCTION

A simple machine may be defined as a device, which enables us to do some useful work at some point or to overcome some resistance, when an effort or force is applied to it, at some other convenient point and a simple lifting machine is a device, which enables us to lift a heavy load (W) by applying a comparatively smaller effort (P).

9.2 BASIC TERMINOLOGIES USED FOR SIMPLE LIFTING MACHINES

9.2.1 MECHANICAL ADVANTAGE

The mechanical advantage (briefly written as M.A.) is the ratio of weight lifted (W) to the effort applied (P) and is always expressed in pure number. Mathematically, mechanical advantage, M.A.= W/P

9.2.2 INPUT OF A MACHINE

The input of a machine is the work done on the machine. In a lifting machine, it is measured by the product of effort and the distance through which it has moved.

9.2.3 OUTPUT OF A MACHINE

The output of a machine is the actual work done by the machine. In a lifting machine, it is measured by the product of the weight lifted and the distance through which it has been lifted.

9.2.4 EFFICIENCY OF A MACHINE

It is the ratio of output to the input of a machine and is generally expressed as a percentage. Mathematically, efficiency,

$$\eta = \{Output/\ Input\}100$$

9.2.5 IDEAL MACHINE

If the efficiency of a machine is 100% i.e., if the output is equal to the input, the machine is called as a perfect or an ideal machine.

9.2.6 VELOCITY RATIO

The velocity ratio (briefly written as V.R.) is the ratio of distance moved by the effort (y) to the distance moved by the load (x) and is always expressed in pure number. Mathematically, velocity ratio,

$$V.R. = y \; / \; x$$

9.3 RELATION BETWEEN EFFICIENCY, MECHANICAL ADVANTAGE AND VELOCITY RATIO OF A LIFTING MACHINE

It is an important relation of a lifting machine, which throws light on its mechanism. Now consider a lifting machine, whose efficiency is required to be found out.

$$\eta = \frac{Output}{Input} = \frac{W \times x}{P \times y} = \frac{W/P}{y/x} = \frac{MA}{VR}$$

9.4 REVERSIBILITY OF A MACHINE

Sometimes, a machine is also capable of doing some work in the reversed direction, after the effort is removed. Such a machine is called a reversible machine and its action is known as reversibility of the machine.

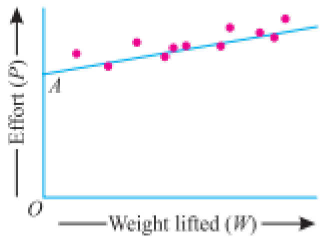
$$\eta > \frac{1}{2} \; ; \; \eta > 50\%$$

Hence the condition for a machine, to be reversible, is that its efficiency should be more than 50%.

9.5 SELF-LOCKING MACHINE

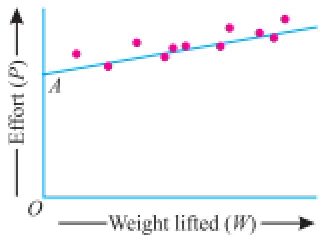
Sometimes, a machine is not capable of doing any work in the reversed direction, after the effort is removed. Such a machine is called a non-reversible or self-locking machine. A little consideration will show, that the condition for a machine to be non-reversible or self-locking is that its efficiency should not be more than 50%.

9.6 FRICTION IN A MACHINE



It has been observed that there is always some amount of friction present in every machine, which can be expressed on a graph of effort (P) and load or weight lifted (W).

9.7 LAW OF A MACHINE



The term 'law of a machine' may be defined as relationship between the effort applied and the load lifted. Thus for any machine, if we record the various efforts required to raise the corresponding loads, and plot a graph between effort and load, we shall get a straight line AB.

P = mW + C where P = Effort applied to lift the load, m = A constant (called coefficient of friction) which is equal to the slope of the line AB.

9.7.1 MAXIMUM MECHANICAL ADVANTAGE OF A LIFTING MACHINE

We know that mechanical advantage of a lifting machine, M.A. W P = For maximum mechanical advantage, substituting the value of P = mW + C in the above equation,

$$Max.\,M.A = \frac{W}{mW+C} = \frac{1}{m+\frac{C}{W}} = \frac{1}{m}$$

9.7.2 MAXIMUM EFFICIENCY OF A LIFTING MACHINE

The maximum efficiency of a lifting machine can be calculated as follows.

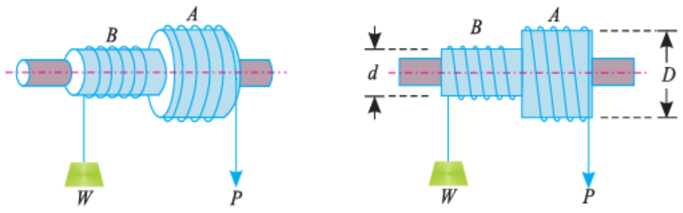
$$Max.\,\eta = \frac{W}{(mW+C)\times VR} = \frac{1}{\left(m+\frac{C}{W}\right)\times VR} = \frac{1}{m\times VR}$$

9.8 SIMPLE WHEEL AND AXLE

A simple wheel and axle, in which the wheel A and axle B are keyed to the same shaft. The shaft is mounted on ball bearings, order to reduce the frictional resistance to a minimum. A string is wounded round the axle B, which carries the load to be lifted. A second string is wound round the wheel A in the opposite direction to that of the string on B.

Let D = Diameter of effort wheel, d = Diameter of the load axle, W = Load lifted, and P = Effort applied to lift the load.

Displacement of the effort in one revolution of effort wheel A, = πD



Displacement of the load in one revolution = πd

$$V.R. = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}} = \frac{\pi D}{\pi d} = \frac{D}{d}$$

$$M.A. = \frac{\text{Load lifted}}{\text{Effort applied}} = \frac{W}{P}$$

$$\eta = \frac{M.A}{V.R.}$$

9.9 PULLEY SYSTEMS

A simple pulley is a wheel of metal or wood, with a groove around its circumference, to receive rope or chain. The pulley rotates freely about its axle, which passes through its centre and is perpendicular to its surface plane. This axle is supported by a metal or a wooden frame, called block as show in Fig. The following assumptions are made in the study of pulley system, which are quite reasonable from the practical point of view :

1. The weight of the pulley block is small as compared to the weight to be lifted, and thus may be neglected in calculations.

2. The friction between the pulley surface and the string is negligible, and thus the tension in the two sides of the rope, passing round the pulley, may be taken to be equal. A little consideration will show, that in a simple pulley, its mechanical advantage as well as velocity ratio is 1 under the assumed conditions mentioned above.

9.9.1 TYPES OF PULLEY SYSTEMS

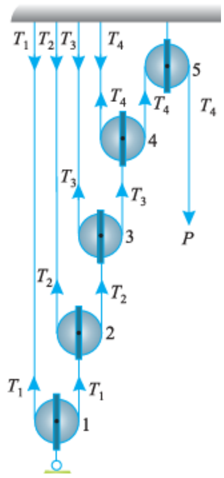
Following three types of pulley systems are commonly used.

1. First system of pulleys.

2. Second system of pulleys.

3. Third system of pulleys

9.9.1.1 FIRST ORDER SYSTEM OF PULLEYS



The fig. shows a first order pulley system. It comprises of fixed and moveable pulleys. Pulleys 1,2,3, and 4 are moveable pulleys whereas pulley no. 5 is a fixed pulley.

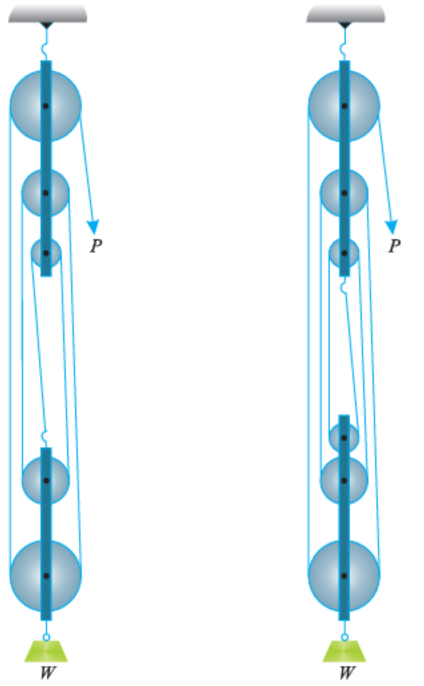
VR= 2^n, where n is no. of moveable pulleys.

So, in this case VR=16

9.9.1.2 SECOND ORDER SYSTEM OF PULLEYS

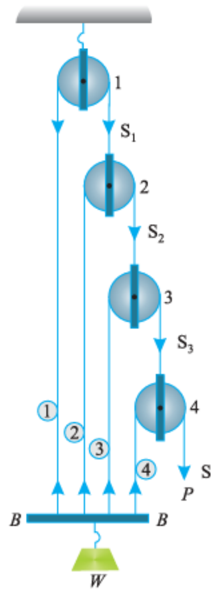
The figs. a and b shows second order pulley system.

The Velocity ratio, VR = n where n is no. of pulleys in both the blocks.



(a) Different no. of pulleys (b) Same no. of pulleys

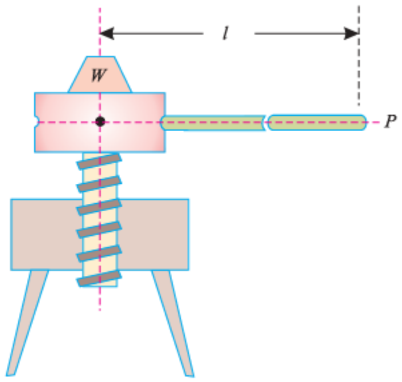
9.9.1.3 THIRD ORDER SYSTEM OF PULLEYS



The velocity ratio, of the system, may be obtained by considering a unit motion of the load.

The VR= 2^n-1, where n is total no. of pulleys.

9.10 SIMPLE SCREW JACK



It consists of a screw, fitted in a nut, which forms the body of the jack. The principle, on which a screw jack works, is similar to that of an inclined plane.

A simple screw jack is shown in fig., which is rotated by the application of an effort at the end of the lever, for lifting the load. Now consider a single threaded simple screw jack.

Let l = Length of the effort arm,

p = Pitch of the screw,

W = Load lifted, and

P = Effort applied to lift the load at the end of the lever.

Distance moved by the effort in one revolution of screw, = 2πl

Distance moved by the load = p

$$VR= 2\pi l/p$$

$$P = W \tan (\alpha + \varphi)$$

where , W is load to be lifited

and tan α=p/πd

tan φ = μ = Coefficient of friction.

Sample Problems

1. The law of a certain lifting machine is : P= W/50+ 8,

The velocity ratio of the machine is 100. Find the maximum possible mechanical advantage and the maximum possible efficiency of the machine. Determine the effort required to overcome the machine friction, while lifting a load of 600 N. Also calculate the efficiency of the machine at this load.

Solution. Given: Law of lifting machine $P = \frac{W}{50} + 8 = 0.02W + 8$; Velocity ratio (V.R.) = 100 and load (W) = 600 N.

Maximum possible mechanical advantage

Comparing the given law of the machine with the standard relation for the law of the machine (*i.e.* $P = mW + C$) we find that in the given law of the machine, $m = 0.02$. We know that maximum possible mechanical advantage

$$\text{Max M.A.} = \frac{1}{m} = \frac{1}{0.02} = 50 \quad \textbf{Ans.}$$

Maximum possible efficiency

We know that maximum possible efficiency

$$= \frac{1}{m \times \text{V.R.}} = \frac{1}{0.02 \times 100} = \frac{1}{2} = 0.5 = 50\% \quad \textbf{Ans.}$$

Effort required to overcome the machine friction

We know that effort required to lift a load of 600 N

$$P = mW + 8 = (0.02 \times 600) + 8 = 20 \text{ N}$$

and effort required to overcome the machine friction, while lifting a load of 600 N,

$$F_{(\text{effort})} = P - \frac{W}{\text{V.R.}} = 20 - \frac{600}{100} = 14 \text{ N} \quad \textbf{Ans.}$$

Efficiency of the machine

We know that mechanical advantage of the machine while lifting a load of 600 N

$$\text{M.A} = \frac{W}{P} = \frac{600}{20} = 30$$

and efficiency,

$$\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{30}{100} = 0.3 = 30\% \quad \textbf{Ans.}$$

2. In a third system of pulleys, there are 4 pulleys. Find the effort required to lift a load of 1800 N, if efficiency of the machine is 75%. Calculate the amount of effort wasted in friction.

Solution. Given: No. of pulleys $(n) = 4$; Load lifted $(W) = 1800$ N and efficiency $(\eta) = 75\% = 0.75$.

Effort required to lift the load

Let P = Effort required in newton to lift the load.

We know that velocity ratio of third system of pulleys.

$$\text{V.R.} = 2^n - 1 = 2^4 - 1 = 15$$

and

$$\text{M.A.} = \frac{W}{P} = \frac{1800}{P}$$

We also know that efficiency

$$0.75 = \frac{\text{M.A.}}{\text{V.R.}} = \frac{\frac{1800}{P}}{15} = \frac{120}{P}$$

or

$$P = \frac{120}{0.75} = 160 \text{ N} \qquad \textbf{Ans.}$$

Effort wasted in friction

We know that effort wasted in friction,

$$F_{(\text{effort})} = P - \frac{W}{\text{V.R.}} = 160 - \frac{1800}{15} = 40 \text{ N} \qquad \textbf{Ans.}$$

CHAPTER 10 KINETICS OF A PARTICLE

Course content
Types of motion, linear motion with uniform velocity, uniform & varying acceleration, motion under gravity, motion of projectiles, concept of relative and resultant velocity. Newton's laws of motion, equation of motion for system of particles, D' Alembet's Principle, Motion of connecting bodies. Concept of momentum, Impulse momentum, Conservation of momentum and energy, Principle of work and energy

10.1 INTRODUCTION

A body is said to be at rest, if it occupies the same position with respect to its surroundings at all moments. But it is said to be in motion, if it changes its position, with respect to its surroundings.

10.2 PARAMETERS DEFINING MOTION

1. Speed: The speed of a body may be defined as its rate of change of displacement with respect to its surroundings. The speed of a body is irrespective of its direction and is, thus, a scalar quantity.
2. Velocity: The velocity of a body may be defined as its rate of change of displacement, with respect to its surroundings, in a particular direction. As the velocity is always expressed in particular direction, therefore it is a vector quantity.
3. Acceleration: The acceleration of a body may be defined as the rate of change of its velocity. It is said to be positive, when the velocity of a body increases with time, and negative when the velocity decreases with time. The negative acceleration is also called retardation. In general, the term acceleration is used to denote the rate at which the velocity is changing. It may be uniform or variable.
4. Uniform acceleration: If a body moves in such a way that its velocity changes in equal magnitudes in equal intervals of time, it is said to be moving with a uniform acceleration.

5. Variable acceleration: If a body moves in such a way, that its velocity changes in unequal magnitudes in equal intervals of time, it is said to be moving with a variable acceleration.
6. Distance traversed: It is the total distance moved by a body. Mathematically, if body is moving with a uniform velocity (v), then in (t) seconds, the distance traversed

$$s = vt$$

10.3 MOTION UNDER UNIFORM ACCELERATION

The following equations defines the motion under uniform acceleration.

$$v = u \pm a\,t$$

$$s = \text{Average velocity} \times \text{Time}$$

$$= \{u+v\} \text{ x } t/2$$

$$S = ut \pm 1/2at^2$$

$$v^2 = u^2 \pm 2as$$

10.4 MOTION UNDER FORCE OF GRAVITY

The following equations defines the motion under uniform acceleration.

$$v = u \pm gt$$

$$S = ut \pm 1/2gt^2$$

$$v^2 = u^2 \pm 2gs$$

10.5 MOTION UNDER VARIABLE ACCELERATION

A body, which does not move with a uniform acceleration, is said to be moving with a non-uniform or variable acceleration.

The velocity of a body, is the rate of change of its position.

$$V = ds/dt$$

$$a = d^2S/dt^2 = dV/dt = v.\ dV/ds$$

Sample Problem

1. A particle, starting from rest, moves in a straight line, whose equation of motion is given by : $s = t^3 - 2t^2 + 3$. Find the velocity and acceleration of the particle after 5 seconds.

Solution. Given : Equation of displacement : $s = t^3 - 2t^2 + 3$ ...(*i*)

Velocity after 5 seconds

Differentiating the above equation with respect to *t*,

$$\frac{ds}{dt} = 3t^2 - 4t \quad ...(ii)$$

i.e., velocity, $v = 3t^2 - 4t$ ...$\left(\because \frac{ds}{dt} = \text{Velocity}\right)$

substituting *t* equal to 5 in the above equation,

$$v = 3\,(5)^2 - (4 \times 5) = 55 \text{ m/s}$$ **Ans.**

Acceleration after 5 seconds

Again differentiating equation (*ii*) with respect to *t*,

$$\frac{d^2s}{dt^2} = 6t - 4 \quad ...(iii)$$

i.e. acceleration, $a = 6t - 4$ ...$\left(\because \frac{d^2s}{dt^2} = \text{Acceleration}\right)$

Now substituting *t* equal to 5 in the above equation,

$$a = (6 \times 5) - 4 = 26 \text{ m/s}^2$$ **Ans.**

10.6 RELATIVE VELOCITY

Every motion is relative, as an absolute motion is impossible to conceive. Strictly speaking, our motion is always relative with reference to the Earth , which is supposed to be fixed or at rest. But we know that our Earth is also not at rest.

It has some relative velocity with respect to the celestial bodies such as sun, moon etc. These celestial bodies, in turn, have some relative velocity with respect to the stars of the universe.

10.6.1 METHODS FOR RELATIVE VELOCITY

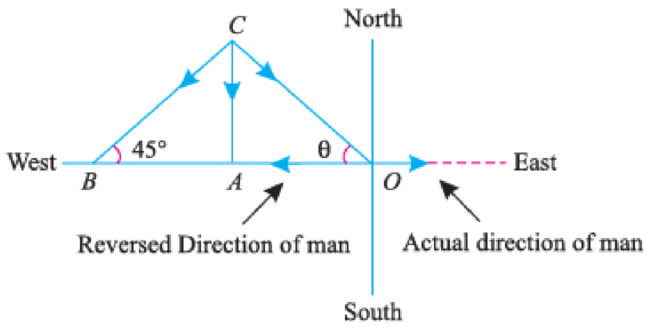
The relative velocity of two bodies may be found out either graphically or analytically.

Sample problem

1. A man, running eastwards with a speed of 6 kilometres per hour, feels the wind to be blowing directly from North. On doubling his speed, he

feels the wind to blow from the North-east. Find the actual direction and velocity of the wind.

Solution. Given : Velocity of man = 6 km.p.h. (East).



We know that

$OA = AB = 6$ km. ...(Speed of man)

and $CA = AB = 6$ km. $\quad \ldots\left(\frac{CA}{AB} = \tan 45^\circ = 1\right)$

∴ $OA = CA$ or $\theta = 45°$ **Ans.**

Now in triangle *OAC*

$$CO = \frac{CA}{\sin 45°} = \frac{6}{0.707} = 8.49 \text{ km.p.h.} \quad \textbf{Ans.}$$

10.7 PROJECTILES

A particle is projected upwards at a certain angle (but not vertical), we find that the particle traces some path in the air and falls on the ground at a point, other than the point of projection. If we study the motion of the particle, we find that the velocity, with which the particle was projected, has two components namely vertical and horizontal.

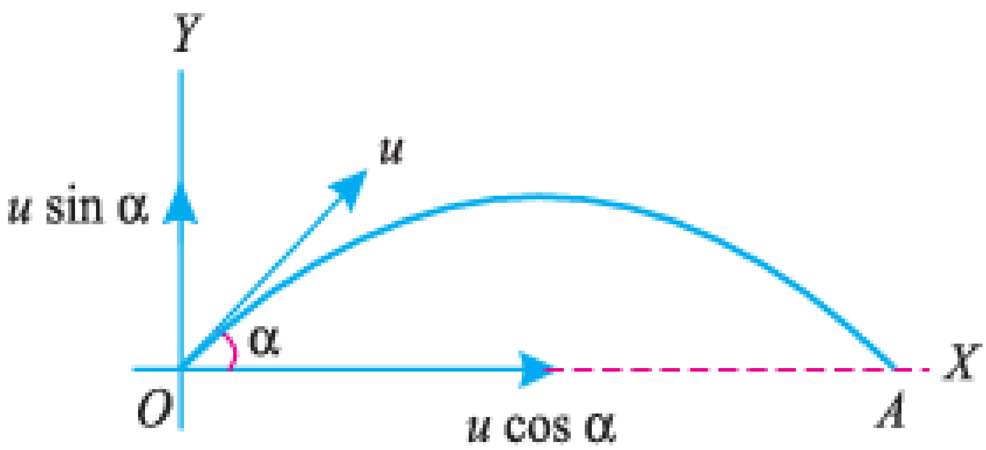
10.7.1 IMPORTANT DEFINITIONS

1. Trajectory: The path, traced by a projectile in the space, is known as trajectory.
2. Velocity of projection: The velocity, with which a projectile is projected, is known as the velocity of projection.
3. Angle of projection: The angle, with the horizontal, at which a projectile is projected, is known as the angle of projection.
4. Time of flight: The total time taken by a projectile, to reach maximum height and to return back to the ground, is known as the time of flight.

5. Range: The distance, between the point of projection and the point where the projectile strikes the ground, is known as the range. It may be noted that the range of a projectile may be horizontal or inclined.

10.8 MOTION OF A PROJECTILE

Consider a particle projected upwards from a point O at an angle α, with the horizontal, with an initial velocity u m/sec as shown in Fig. 20.4. Now resolving this velocity into its vertical and horizontal components, V = u sin α and H = u cos α We know that the vertical component (u sin α) is subjected to retardation due to gravity. The particle will reach maximum height, when the vertical component becomes zero. After this the particle will come down, due to gravity, and this motion will be subjected to acceleration due to gravity.



The horizontal component (u cos α) will remain constant, since there is no acceleration or retardation (neglecting air resistance). The combined effect of the horizontal and the vertical components will be to move the particle, along some path in the air and then the particle falls on the ground at some point A, other than the point of projection O as shown in Fig.

10.8.1 EQUATION OF THE PATH OF A PROJECTILE

Consider a particle projected from a point O at a certain angle with the horizontal. As already discussed, the particle will move along certain path OPA, in the air, and will fall down at A as shown in Fig.

Let u = Velocity of projection, and

α = Angle of projection with the horizontal.

Consider any point P as the position of particle, after t seconds with x and y as co-ordinates as shown in Fig. 20.5. We know that horizontal component of the velocity of projection.

$= u \cos \alpha$

and vertical component $= u \sin \alpha$

$$\therefore \quad y = u \sin \alpha t - \frac{1}{2} g t^2 \qquad ...(i)$$

and $x = u \cos \alpha \, t$

or
$$t = \frac{x}{u \cos \alpha}$$

Substituting the value of t in equation (i),

$$y = u \sin \alpha \left(\frac{x}{u \cos \alpha} \right) - \frac{1}{2} g \left(\frac{x}{u \cos \alpha} \right)^2$$

$$= x \tan \alpha - \frac{g x^2}{2u^2 \cos^2 \alpha} \qquad ...(ii)$$

Since this is the equation of a parabola, therefore path of a projectile (or the equation of trajectroy) is also a parabola.

10.8.2 TIME OF FLIGHT OF A PROJECTILE

$$t = \frac{2u \sin \alpha}{g}$$

10.8.3 HORIZONTAL RANGE OF A PROJECTILE

$$R = \frac{u^2 \sin 2\alpha}{g}$$

10.8.4 MAXIMUM HEIGHT OF A PROJECTILE

$$H = \frac{u^2 \sin^2 \alpha}{2g}$$

Sample Problem

1. If a particle is projected inside a horizontal tunnel which is 5 metres high with a velocity of 60 m/s, find the angle of projection and the greatest possible range.

Solution. Given : Height of the tunnel (H) = 5 m and velocity of projection (u) = 60 m/s.

Angle of projection

Let α = Angle of projection.

We know that height of tunnel (H)

$$5 = \frac{u^2 \sin^2\alpha}{2g} = \frac{(60)^2 \sin^2\alpha}{2 \times 9.8} = 183.7 \sin^2\alpha$$

or

$$\sin^2\alpha = \frac{5}{183.7} = 0.0272$$

$\therefore$ $\sin\alpha = 0.1650$ or $\alpha = 9.5°$ **Ans.**

Greatest possible range

We know that greatest possible range,

$$R = \frac{u^2 \sin 2\alpha}{g} = \frac{(60)^2 \sin(2 \times 9.5°)}{9.8} = \frac{(60)^2 \sin 19°}{9.8} \text{ m}$$

$$= \frac{3600 \times 0.3256}{9.8} = 119.6 \text{ m}$$ **Ans.**

10.9 NEWTON'S LAWS OF MOTION

1. Newton's First Law of Motion states, "Everybody continues in its state of rest or of uniform motion, in a straight line, unless it is acted upon by some external force."

2. Newton's Second Law of Motion states, "The rate of change of momentum is directly proportional to the impressed force, and takes place in the same direction, in which the force acts."

3. Newton's Third Law of Motion states, "To every action, there is always an equal and opposite reaction."

10.10 D'ALEMBERT'S PRINCIPLE

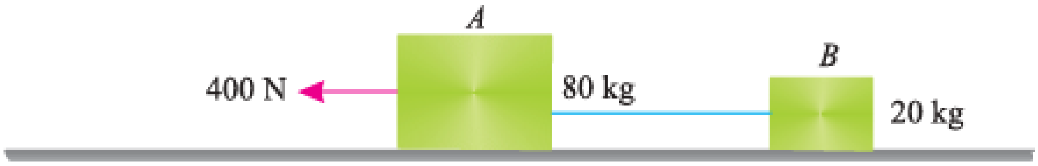
It states, "If a rigid body is acted upon by a system of forces, this system may be reduced to a single resultant force whose magnitude, direction and the line of action may be found out by the methods of graphic statics."

$$R - ma = 0$$

Sample Problem

1. Two bodies A and B of mass 80 kg and 20 kg are connected by a thread and move along a rough horizontal plane under the action of a force 400 N applied to the first body of mass 80 kg as shown in Fig. The coefficient of friction between the sliding surfaces of the bodies and the

plane is 0.3. Determine the acceleration of the two bodies and the tension in the thread, using D' Alembert's principle.

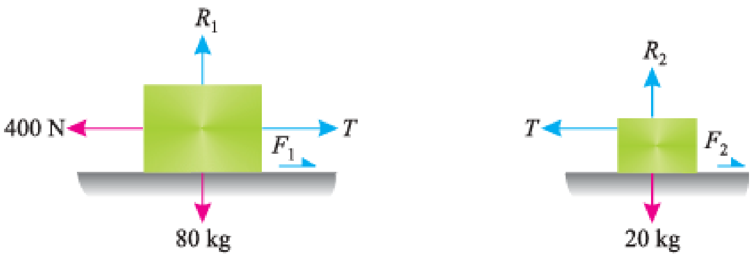


Solution. Given : Mass of body A (m_1) = 80 kg ; Mass of the body B (m_2) = 20 kg; Force applied on first body (P) = 400 N and coefficient of friction (μ) = 0.3

Acceleration of the two bodies

Let a = Acceleration of the bodies, and

T = Tension in the thread.



(*a*) *Body A* (*b*) *Body B*

Fig. 24.3.

First of all, consider the body A. The forces acting on it are :

1. 400 N force (acting towards left)
2. Mass of the body = 80 kg (acting downwards)
3. Reaction $R_1 = 80 \times 9.8 = 784$ N (acting upwards)
4. Force of friction, $F_1 = \mu R_1 = 0.3 \times 784 = 235.2$ N (acting towards right)
5. Tension in the thread = T (acting towards right).

∴ Resultant horizontal force,

$$P_1 = 400 - T - F_1 = 400 - T - 235.2$$
$$= 164.8 - T \text{ (acting towards left)}$$

We know that force causing acceleration to the body A

$$= m_1 a = 80\, a$$

and according to D' Alembert's principle $(P_1 - m_1 a = 0)$

$$164.8 - T - 80\, a = 0$$

or $$T = 164.8 - 80a \qquad ...(i)$$

Now consider the body B. The forces acting on it are :

1. Tension in the thread = T (acting towards left)
2. Mass of the body = 20 kg (acting downwards)

3. Reaction $R_2 = 20 \times 9.8 = 196$ N (acting upwards)

4. Force of friction, $F_2 = \mu R_2 = 0.3 \times 196 = 58.8$ N (acting towards right)

∴ Resulting horizontal force,

$$P_2 = T - F_2 = T - 58.8$$

We know that force causing acceleration to the body *B*

$$= m_2 a = 20\,a$$

and according to *D′* Alembert's principle ($P_2 - m_2 a = 0$)

$$(T - 58.8) - 20\,a = 0$$

or $$T = 58.8 + 20\,a$$

Now equating the two values of *T* from equation (*i*) and (*ii*),

$$164.8 - 80\,a = 58.8 + 20\,a$$

$$100\,a = 106$$

or $$a = \frac{106}{100} = 1.06 \text{ m/s}^2$$ **Ans.**

Tension in the thread

Substituting the value of *a* in equation (*ii*),

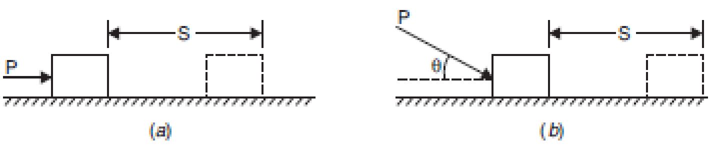
$$T = 58.8 + (20 \times 1.06) = 80 \text{ N}$$ **Ans.**

10.11 WORK AND ENERGY

Work is defined as the product of force and displacement of the body on which force is acting. The force and displacement should be in the same direction. *Energy* is defined as the capacity to do work. The energy exists in many forms like, mechanical, electrical, heat, chemical and light etc. But in engineering mechanics, we only consider mechanical energy.

10.11.1 WORK

work is the product of force and distance. The distance should be in the direction of the force. If a force *P* is acting on a body and the body moves a distance of *S* in the direction of the force, as shown in Fig.



Work done = Force × Distance = P × S

But if the force acting on the body and the distance moved are not in the same direction. then the work done on the body is given by;

Work done = Component of force in the direction of motion × Distance

$$= P \cos \theta \times S$$

10.11.2 ENERGY

The capacity of doing work is known as energy. It is the product of power and time. The energy is expressed in Nm. It exists in many forms i.e., mechanical, electrical, heat, chemical, light etc. In engineering mechanics, we are only concerned with mechanical energy and the same will be dealt with.

(*i*) Potential energy (or position energy or datum energy)

(*ii*) Kinetic energy

Potential energy is also known as position energy or datum energy. It is the energy by *virtue of position* of a body with respect to any given reference or datum. The energy possessed by a body by virtue of its velocity (or its motion) is known as kinetic energy. It is represented by K.E.

10.11.3 PRINCIPLE OF WORK AND ENERGY

It states that the change in kinetic energy of a body during any displacement is equal to the work done by the net force acting on the body. Or we can say that work done is equal to change of kinetic energy of the body.

$$F.\ ds = K.E. = \frac{1}{2} m v^2 = m \times v\ dv$$

10.12 LINEAR IMPULSE AND MOMENTUM

Impulse-momentum method is also used. This method which relates force, velocity and time, is based on the integration of equation of motion with respect to time.

$$F \cdot dt = m \cdot dv$$

$$\int_{t_1}^{t_2} F \times dt = \int_{v_1}^{v_2} m \times dv$$

10.13 CONSERVATION OF LINEAR MOMENTUM

The momentum of a system in linear direction is always conserved. This means the momentum of a system remains constant. Or in other words the total momentum of a system in a direction before collision is equal to the total momentum of the system after collision in that direction.

Final momentum = Initial momentum

$$M_2V_2 - M_1V_1 = M_2U_2 - M_1U_1$$

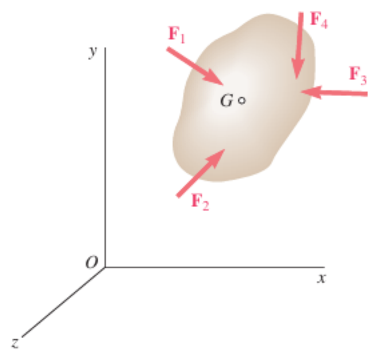
CHAPTER 11 KINETICS OF A RIGID BODY

Course content
Introduction, Equation of motion for a rigid body, Angular Momentum of Rigid Bodies, D'Alembert's principle applied to bodies having linear and angular motion. Equation of dynamic equilibrium.

11.1 INTRODUCTION

Kinetics is that branch of Engineering Mechanics which deals with the force system which produces acceleration and resulting motion of bodies. In this chapter the motion of a rigid body is considered to be a combination of translation of the body and rotation about its mass center and then the analysis will be carried out regarding the relationship between forces and moments acting on a rigid body and the body's linear and angular acceleration.

11.2 EQUATIONS OF MOTION FOR A RIGID BODY



Consider a rigid body acted upon by several external forces F1, F2, F3, ... We can assume that the body is made of a large number n of particles of mass Δmi (i = 1, 2, . . . , n). Consider first the motion of the mass center G of the body with respect to the Newtonian frame of reference O_{xyz}.

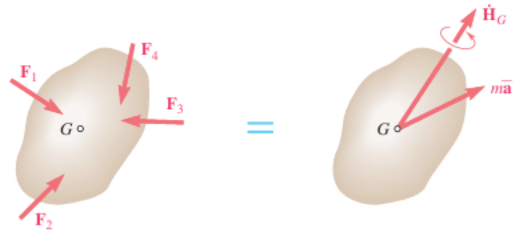
Equation of Translatory motion

$\Sigma F = m\bar{a}$, where m is the mass of the body and $\bar{a}$ is the acceleration of the mass center G. Turning now to the motion of the body relative to the centroidal frame of reference Gx′y′z′.

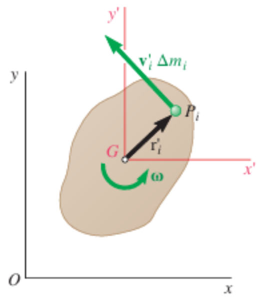
Equation of Rotational motion

$\Sigma M_G = \dot{\boldsymbol{H}}_{\boldsymbol{G}}$, where $\dot{\boldsymbol{H}}_{\boldsymbol{G}}$ represents the rate of change of H_G, which is the angular momentum about G of the system of particles forming the rigid body. In the following discussion, we refer to H_G simply as the angular momentum of the rigid body about its mass center G.

The system of the external forces and moments is equipollent to the system consisting of the vector $m\bar{a}$ attached at G and the couple of moment $\dot{\boldsymbol{H}}_{\mathbf{G}}$.



11.3 ANGULAR MOMENTUM OF A RIGID BODY IN PLANE MOTION



Consider a rigid body in plane motion. Assume that the body is made of a large number n of particles Pi with a mass Δmi. The angular momentum H_G of the rigid body about its mass center G by taking the moments about G of the momenta of the particles of the body with respect to either of the frames Oxy or Gx'y, is given by

$H_G = \sum_{i=j}^{n} (r_j' \times v_i' \Delta m_i)$), this expression represents a vector of the same direction as ω (i.e., perpendicular to the body) and with a magnitude of ωΣ $r_{i'}^2 \Delta m_i$.

Angular momentum of a rigid body about G

$H_G = \bar{I}\omega$

Rate of change of angular momentum about G

$\dot{H}_G = \bar{I}\dot{\omega} = \bar{I}a$, Thus, the rate of change of the angular momentum of the rigid body is represented by a vector in the same direction as α (i.e., perpendicular to the body) with a magnitude $\bar{I}a$.

11.4 D' ALEMBERT'S PRINCIPLE APPLICABLE TO PLANE MOTION

Let a body of mass 'm' be moving with a uniform acceleration 'a' under the action of external force F (also known as effective force).

According to Newton's seconds law of motion, we have F =m × a

The above equation can also be written as F–ma = 0 it is clear that by applying a force **(– ma)** on the body, the body will be in equilibrium as the sum of all forces acting on the body is zero. Such equilibrium is called **dynamic equilibrium** and the force **(– ma)** is called D' Alembert force or inertia force.

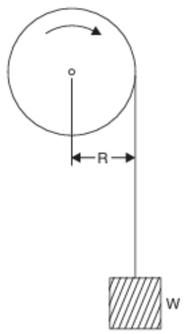
Thus, the body will be in dynamic equilibrium under the action of external force F and the inertia force. This is known as D' Alembert's Principle.

11.5 D' ALEMBERT'S PRINCIPLE APPLICABLE TO ROTARY MOTION

It states that when external torques (also called active torques) acts on a system having rotating motion, then the algebraic sum of all the torques acting on the system due to external forces and reversed active forces including the inertia torques (taken in the opposite direction of the angular acceleration) is zero.

Case I: Rotation due to a weight W attached to one end of a string, passing over a pulley of weight W_0.

A weight W is attached to one end of an inextensible string, which passes over a pulley of weight W0, the other end of the string is attached to the periphery of the pulley as shown in Fig. When the weight W moves downwards, the rotation to the pulley is caused in clockwise direction.



Let

a= Linear acceleration of the weight (W)= αR

α=Angular acceleration of the pulley

R=Radius of the pulley

I =Moment of inertia of the pulley about the axis of rotation.

Inertia force on weight W = **- {W/g} x a= - {W/g} x αR**

Now torque due to external force = W × R

Torque due to inertia force = inertia × R= **- {W/g} x αRx R=- {W/g} x αR^2**

Torque on the pulley due to angular acceleration =I × α

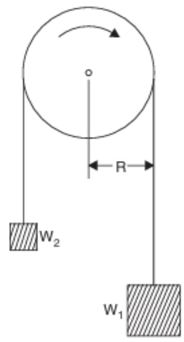
Inertia torque, taken in opposite direction of the angular acceleration =– I × α

According to D' Alembert's Principle

$W \times R - \{W/g\} \times \alpha R^2 - I \times \alpha = 0$

$$= \quad \alpha = \left(\frac{W \times R}{\frac{W}{g} R^2 + I} \right)$$

Case II: Rotation due to weights attached to the two ends of a string, which passes over a rough pulley of weight W_0.



Let

R=Radius of pulley

a=Linear acceleration of the system

α=Angular acceleration of the pulley W_1,

W_2=Two weights attached

W_0=Weight of pulley

I=Moment of inertia of the pulley $=MR^2/2$

if pulley is considered a solid disc = $\{W_0/g\} R^2/2$

Consider W1 > W2

Resultant torque caused by the weights

$= W_1 \times R - W_2R = (W_1 - W_2) \times R$

Inertia force on weight W_1

$=- \{ W_1/g\}$ a

Torque due to inertia force on weight W_1

$=- \{ W_1/g\} \alpha R$

Similarly, the intertia force and torque on W_2 can be calculated'

Intertia torque $=- I \times \alpha$

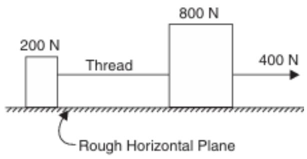
According to D' Alembert's Principle,

$(W_1 - W_2) \times R - \{ W_1/g\} \alpha R - \{ W_2/g\} \alpha R - I \times \alpha = 0$

These above equations are also referred to as the equations of dynamic equilibrium.

Sample Problem

1. Two weights 800 N and 200 N are connected by a thread and move along a rough horizontal plane under the action of a force 400 N applied to the first weight of 800 N as shown in Fig. The co-efficient of friction between the sliding surfaces of the weights and the plane is 0.3. Determine the acceleration of the weights and the tension in the thread using D' Alembert's principle.



Given:

W1=200 N

W2=800 N

Force applied, F=400 N

Co-efficient of friction,μ=0.3

Let a=Acceleration of weights

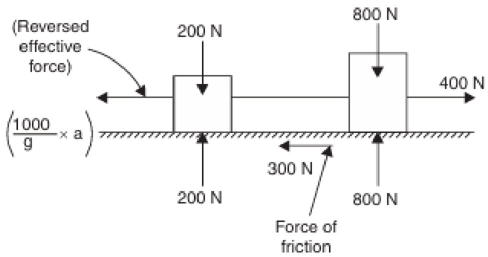
T=Tension in the threads.

Total weight of the system = W1 + W2 = 200 + 800 = 1000 N

Total mass of the system = Total weight = 1000/g kg

Inertia force on the system = – (Mass of system) × acceleration of the system = – 1000/g × a Newton

Force of friction on the system = μ × Total weight = 0.3 × (1000) = 300N



Force applied – Force of friction + Inertia force = 0

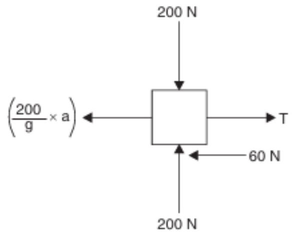
400 – 300 – 1000/g × a = 0

100 = 1000×a/ g

a= 100/1000 ×g = g/10 = 9.8/10 = 0.98 m/s^2

Tension in the thread

Let T = Tension in the thread Consider the motion of 200 N weight. The free body diagram of 200 N weight is shown.



Force of friction = μ × 200 = 0.3 × 200 = 60 N

Inertia force on it = Mass × Acceleration = 200/g × a = 200/g × g/10 = 20 N

By D' Alembert's principle,

T – 60 – 20 = 0 ∴ T = 60 + 20 = 80 N